EGR326 F'09

Assignment #2

Solutions

Analytical Exercises

1. Lecture Notes Chapter 2 Exercise #9 (page 94).

   (a) You know that voltage divider regulators are extremely inefficient and have very poor load regulation, especially at currents as high as 1A. Let's discount that option and move on to the next least expensive option, the shunt regulator. We could use a 5V zener diode (if one can be found) or an LM431 programmed for 5V operation. The resistor component of the shunt regulator must be designed for minimum input voltage:

   $$R = \frac{7\text{V} - 5\text{V}}{1\text{A}} = 2\Omega$$

   At an input voltage of 9V, however, the efficiency of the regulator is:

   $$\begin{aligned} \eta &= \frac{P_{out}}{P_{in}} \\ &= \frac{5\text{V} \cdot 1\text{A}}{9\text{V} \cdot \left(\frac{9-5}{2}\right)} \\ &= 27.8\% \end{aligned}$$

   which does not meet the 50% efficiency requirement.

   Let's move on to the next least expensive option, the linear regulator (remember...you can get a 7805 for about \$0.30). The worst-case efficiency of this regulator occurs at maximum input voltage:

   $$\begin{aligned} \eta &= \frac{5\text{V} \cdot 1\text{A}}{9\text{V} \cdot 1\text{A}} \\ &= 55.6\% \end{aligned}$$

   Thus, the linear regulator is the least expensive option that maintains at least 50% efficiency for an input voltage of 7V-9V and a load current of 1A.

   (b) With such a low load current (10μA) it is entirely reasonable to consider a voltage divider. We need to choose R_1 and R_2 in a voltage divider circuit such that:

   $$\begin{aligned} 1.24\text{V} &= 5\text{V}\frac{R_2}{R_1 + R_2} \\ 0.248\left(R_1 + R_2\right) &= R_2 \\ 0.248R_1 &= 0.752R_2 \\ R_1 &= 3.03R_2 \end{aligned}$$

   Commercially available 1% precision resistors of values $R_2 = 1.02\text{k}$ and $R_1 = 3.09\text{k}$ give a no-load output voltage of 1.2409V and draw only 1.2mA from the 5V power supply in the resistor divider configuration. With a 10μA load, the output voltage becomes (see page 3 of your lecture notes):

$$
\begin{aligned}
V_{out} &= \frac{R_2}{R_1 + R_2} \cdot V_{in} - (R_1 \parallel R_2) \cdot I_{out} \\
&= \frac{1020}{1020 + 3090} \cdot 5 - (1020 \parallel 3090) \cdot 10^{-5} \\
&= 1.233\text{V}
\end{aligned}
$$

This is only 8mV lower than the no-load voltage. Perhaps this load regulation performance is good enough? If not, you can use resistors that are smaller for better load regulation, but more current draw from the power supply.

Without any constraints on current draw or efficiency, or accuracy constraints on the 1.24V output, there isn't any reason to go further.

(c) This is very similar to part (a), so let's pick up where that problem left off, with a linear regulator. The worst-case efficiency is at the maximum input voltage:

$$
\begin{aligned}
\eta &= \frac{5\text{V}}{12\text{V}} \\
&= 41.7\%
\end{aligned}
$$

which is not good enough. We must use a switching regulator (step-down converter) to achieve efficiencies of 85% or higher.

(d) Let's start from the beginning with a voltage divider, the least expensive option. A voltage output of 2.8V ± 10% means we can have an output voltage as high as 2.8V + 10% = 3.08V at no load current and as low as 2.8V − 10% = 2.52V at the maximum load current of 100mA. This means that in the equation defining output voltage:

$$
V_{out} = \frac{R_2}{R_1 + R_2} \cdot V_{in} - (R_1 \parallel R_2) \cdot I_{out}
$$

we must have:

$$
(R_1 \parallel R_2) \leq \frac{3.08\text{V} - 2.52\text{V}}{0.1\text{A}}
$$

In other words, the slope of the straight-line equation for V_{out} must be no steeper than the maximum change in output voltage over a 100mA range. Following the inequality:

$$
\begin{aligned}
\frac{R_1 R_2}{R_1 + R_2} &\leq \frac{0.56}{0.1} \\
R_1 R_2 &\leq 5.6\,(R_1 + R_2)
\end{aligned}
$$

That can be our first constraint. Our second constraint is set by the need for an output voltage of 3.08V at no load current:

$$
\begin{aligned}
3.08\text{V} &= 5\text{V} \cdot \frac{R_2}{R_1 + R_2} \\
0.616\,(R_1 + R_2) &= R_2 \\
0.616 R_1 &= 0.384 R_2 \\
R_1 &= 0.623 R_2
\end{aligned}
$$

Substituting this into our first constraint:

$$\begin{aligned} R_1 R_2 &\leq 5.6\,(R_1 + R_2) \\ 0.623 R_2^2 &\leq 5.6\,(0.623 R_2 + R_2) \\ 0.623 R_2^2 &\leq 9.09 R_2 \\ R_2 &\leq 14.6\Omega \end{aligned}$$

Let's choose $R_2 = 14.6\Omega$ (assuming we can find such a resistor) which requires that $R_1 = 9.1\Omega$. What is the power dissipation of this voltage divider? At no load, the power dissipation is:

$$\begin{aligned} P &= \frac{V^2}{R} \\ &= \frac{5^2}{R_1 + R_2} \\ &= \frac{25}{14.6 + 9.1} \\ &= 1.05\text{W} \end{aligned}$$

As expected, voltage dividers are fairly wasteful of power, especially when reasonably good load regulation is required. The voltage divider clearly doesn't meet the design upper bound of 0.6W of power dissipation in the regulator, so we must dismiss the voltage divider.

Moving on to the shunt regulator, the next least expensive option, we must either find a 2.8V zener diode or use an LM431 circuit programmed for 2.8V operation. The resistor in the shunt regulator must be chosen to deliver 101mA of current (100mA for the load and 1mA to bias the zener diode) at an input voltage of 5V. Whatever resistor we choose it will always dissipate:

$$P_R = V \cdot I = (5 - 2.8) \cdot 0.101 = 0.222\text{W}$$

and the zener diode will dissipate:

$$P_Z = V \cdot I = 2.8 \cdot 0.101 = 0.283\text{W}$$

when, in the worst case, there is no load current and all 101mA of current is shunted by the zener diode.

The combined power dissipation in the regulator is 0.222W+0.238W or 0.505W which is within our 0.6W design limit. The shunt regulator, then, is an acceptable solution, as long as our zener diode solution maintains the 2.8V $\pm$ 10% accuracy requirement. A precision device such as the LM431 will easily fulfill this requirement.

(e) This one is simple...only a switching regulator is capable of increasing an input voltage of 4V-6V to the higher potential of 8.2V. The remaining three solutions are only capable of reducing a higher voltage to a lower voltage.