EGR326 F'09

Assignment #1

Solutions

Analytical Exercises

1. Lecture Notes Chapter 2 Exercise #1 (page 94).

   (a) A 12V zener diode (1N4742 or equivalent) should be used. To provide 50mA (always) of load current and reserve 1mA for biasing the zener diode (Fairchild's datasheet, for example, only guarantees 11.4V $\leq V_z \leq$ 12.6V for I_z = 21mA but the actual current I_z is not as important as the design procedure), we choose the resistor as follows:

$$\begin{aligned} R &= \frac{V_{in} - V_{out}}{I} \\ &= \frac{15 - 12}{0.051} \\ &= 58.8\Omega \end{aligned}$$

   A commercially-available 57.6Ω resistor should work.

   Best-case efficiency and worst-case efficiency will be the same since there is no variation in either input voltage or output voltage. The efficiency will be:

$$\begin{aligned} \eta &= \frac{P_{out}}{P_{in}} \\ &= \frac{12 \cdot 0.05}{15 \cdot 0.051} \\ &= 78.4\% \end{aligned}$$

   (b) Following the same reasoning as part (a), we select:

$$\begin{aligned} R &= \frac{V_{in} - V_{out}}{I} \\ &= \frac{14 - 12}{0.051} \\ &= 39.2\Omega \end{aligned}$$

   Best-case efficiency will occur at maximum load current and minimum input voltage:

$$\begin{aligned} \eta &= \frac{12 \cdot 0.05}{14 \cdot 0.051} \\ &= 84\% \end{aligned}$$

   Worst-case efficiency will occur at minimum load current and maximum input voltage. Since the minimum load current is 0, the efficiency will clearly be 0%.

   (c) Following the same reasoning, a 15V zener diode should be used (MMBZ5245 or equivalent) and the resistor chosen so that:

$$\begin{aligned} R &= \frac{V_{in-min} - V_{out}}{I_{load-max} + I_z} \\ &= \frac{20 - 15}{0.05 + 0.01} \\ &= 98\Omega \end{aligned}$$

Note that the *minimum* input voltage and *maximum* load current were used in computing the required value of R – refer back to Section 1.2 to see why (or just think about it).

A 97.6Ω resistor can be used (don't believe me? Go to www.digikey.com and look up part number RHM97.6FCT-ND).

Worst-case efficiency will occur at maximum input power and minimum output power. This will occur at maximum input voltage and minimum load current:

$$\eta_{min} = \frac{P_{out}}{P_{in}} = \frac{15 \cdot 0.025}{28 \cdot \frac{(28-15)}{97.6}} = 10\%$$

Best-case efficiency occurs at minimum input voltage and maximum load current:

$$\eta_{max} = \frac{P_{out}}{P_{in}} = \frac{15 * 0.05}{20 \cdot \left(\frac{20-15}{97.6}\right)} = 73.2\%$$

(d) We select a 5.1V zener diode (commonly available) and choose the resistor according to maximum load current and minimum input voltage:

$$\begin{aligned} R &= \frac{V_{in} - V_{out}}{I} \\ &= \frac{9 - 5.1}{0.015 + 0.001} \\ &= 243.8\Omega \end{aligned}$$

Maximum efficiency occurs at maximum load current and minimum input voltage:

$$\begin{aligned} \eta &= \frac{5.1 \cdot 0.015}{9 \cdot 0.016} \\ &= 53\% \end{aligned}$$

Minimum efficiency will once again be zero since minimum load current is 0.

(e) Once again we choose R for minimum input voltage and maximum load current, reserving 1mA for the 5.1V zener diode:

$$R = \frac{6 - 5.1}{0.016} = 56.25\Omega$$

A 56Ω resistor can be used. The worst-case efficiency is 0% and the best case efficiency is given by minimum input voltage and maximum load current:

$$\eta = \frac{5.1 \cdot 0.015}{6 \cdot \left(\frac{6-5.1}{56}\right)} = 79.3\%$$

2. Lecture Notes Chapter 2 Exercise #2.

   Generalizing the approach of Problem #1, the design procedure for the one and only thing we can really design in a shunt regulator, the resistor, is to select it based on minimum input voltage and maximum load current, plus some bias current for the zener diode (let's keep using 1mA).

$$\begin{aligned} R &= \frac{V_{min} - V_{out}}{I_{max} + 0.001} \\ &\approx \frac{V_{min} - V_{out}}{I_{max}} \end{aligned}$$

   for simplicity. The worst-case power dissipation for the resistor occurs at maximum input voltage:

$$\begin{aligned} P_R &= \frac{V^2}{R} \\ &= \frac{(V_{max} - V_{out})^2}{\left(\frac{V_{min} - V_{out}}{I_{max}}\right)} \\ &= \frac{I_{max}(V_{max} - V_{out})^2}{V_{min} - V_{out}} \end{aligned}$$

   The diode's worst-case power dissipation also occurs at minimum load current and maximum input voltage, as most of the current from the source flows through the zener diode:

$$\begin{aligned} P_D &= VI_D \\ &= V(I_{in} - I_{load}) \\ &= V_{out}\left(\frac{V_{max} - V_{out}}{R} - I_{min}\right) \\ &= V_{out}\left(\frac{V_{max} - V_{out}}{\left(\frac{V_{min} - V_{out}}{I_{max}}\right)} - I_{min}\right) \\ &= V_{out}\left(\frac{I_{max}(V_{max} - V_{out})}{V_{min} - V_{out}} - I_{min}\right) \end{aligned}$$

   Worst-case efficiency occurs at minimum load current and maximum input voltage:

$$\begin{aligned} \eta_{min} &= \frac{V_{out} \cdot I_{min}}{V_{max} \cdot I_{in}} \\ &= \frac{V_{out} I_{min}}{V_{max}\left(\frac{V_{max} - V_{out}}{R}\right)} \\ &= \frac{V_{out} I_{min}}{V_{max} I_{max} \frac{V_{max} - V_{out}}{V_{min} - V_{out}}} \\ &= \frac{V_{out} I_{min}(V_{min} - V_{out})}{V_{max} I_{max}(V_{max} - V_{out})} \end{aligned}$$

   Finally, best-case efficiency occurs at minimum input voltage and maximum load current:

$$\begin{aligned}\eta_{max} &= \frac{V_{out} I_{max}}{V_{min} I_{in}} \\ &= \frac{V_{out} I_{max}}{V_{min}\left(\frac{V_{min}-V_{out}}{R}\right)} \\ &= \frac{V_{out} I_{max}\left(V_{min}-V_{out}\right)}{V_{min}\left(V_{min}-V_{out}\right) I_{max}} \\ &= \frac{V_{out}}{V_{min}}\end{aligned}$$

The last equation above shows that a shunt regulator can approach the efficiency of a linear regulator in the best case.

3. Lecture Notes Chapter 2 Exercise #3.

   The LM431 tries to maintain its reference terminal to be 2.5V so we have, according to the resistor divider equation:

$$V_{ref} = \frac{R_4}{R_3 + R_4} V_{out} = 2.5\text{V}$$

   Rearranging this to solve for V_{out}:

$$V_{out} = 2.5 \cdot \left(1 + \frac{R_3}{R_4}\right)$$

4. Lecture Notes Chapter 2 Exercise #6.

   The output voltage of a typical LM317 circuit (see Figure 20 on page 19 of your notes) is:

$$V_{out} = 1.25 \cdot \left(1 + \frac{R_2}{R_1}\right)$$

   We want this to be 4.2V, hence we have:

$$\begin{aligned}\frac{4.2}{1.25} &= 1 + \frac{R_2}{R_1} \\ \frac{R_2}{R_1} &= 2.36 \\ R_2 &= 2.36 \cdot R_1\end{aligned}$$

   We want R_1 to provide at least 10mA of load current for the LM317 in case the rest of the circuit isn't drawing any current. Thus, we would like to have:

$$I_{R_1} = \frac{1.25}{R_1} \geq 0.01$$

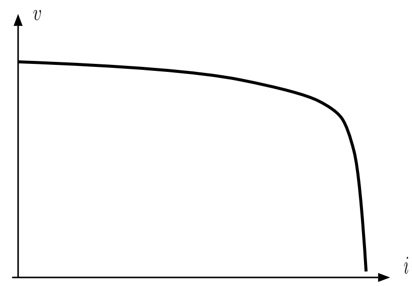
   Using this we solve:

$$R_1 \leq 125\Omega$$

   Let's use $R_1 = 121\Omega$, from which we require $R_2 = 2.36 \cdot R_1 = 285.6\Omega$. A 287Ω resistor would be pretty close...the output voltage would be 4.21V. (Using $R_1 = 113\Omega$ and $R_2 = 267\Omega$ is even better as that gives us $V_{out} = 4.204$V but I doubt that it would make much difference unless ultra-high-precision resistors are used, and the accuracy of the 1.25V reference voltage in the LM317 is also a factor in the overall accuracy).

Laboratory Part I – Shunt Regulator

You should have essentially observed the "third-quadrant" curve of the zener diode, rotated to have current on the X-axis and voltage on the Y-axis.



Laboratory Part II – 7805 Dropout Voltage

You should have observed the output of the 7805 only reach 5V when the input exceeds 6.5V (or so). The datasheet suggests the dropout voltage is 2V, thus you would normally need at least 7V in to get 5V out, but 6.5V (for a dropout voltage of 1.5V) is even better.

We're basically looking for a graph like Figure 14 on page 14 of your lecture notes.

Laboratory Part III – 7805 Accuracy

It's pretty accurate, huh? The 7805 has excellent load regulation so you should have seen a nearly flat line hanging around 5V.

Laboratory Part IV – 7805 Power Dissipation

You should have observed a nearly linear relationship between power dissipation and case temperature, with a slope of about 65^{O}C/W. When we discuss heat transfer later in the course, you will understand why. Above 50^{O}C or so you should have perceived the temperature to be uncomfortably hot. That's only at a power dissipation of about 0.3W-0.4W, well within the power dissipation limits of the 7805. Moral: components that are getting hot could be a sign of excessive current draw, but the human finger is not necessarily a good measure of "hot".

The ideal linear model at a room temperature of 21^{O}C would be:

$$Temp = 65 \cdot Power + 21$$

Some variation in these numbers is expected, of course. To compute maximum allowable input voltage, we begin with the constraint that junction temperature should not exceed 125^{O}C:

$$\begin{aligned} T_{case} &= T_{junction} - P_d \cdot \theta_{JC} \\ T_{junction} &= T_{case} + P_d \cdot \theta_{JC} \leq 125 \end{aligned}$$

Substituting in our experimentally-determined linear equation for case temperature:

$$\begin{aligned} (65 \cdot P_d + 21) + P_d \cdot \theta_{JC} &\leq 125 \\ (65 + \theta_{JC})\, P_d + 21 &\leq 125 \\ P_d &\leq \frac{104}{65 + \theta_{JC}} \end{aligned}$$

The 7805 datasheet (for the TO-220 package) lists $\theta_{JC} = 5^{\text{O}}$C/W (that's a property of any TO-220 package) which gives:

$$P_d \leq \frac{104}{70} = 1.49\text{W}$$

The power dissipation of the 7805 regulator is, ignoring the bias current flowing out of the ground pin, is:

$$P_d = (V_i - V_o) \cdot I_{load}$$

With $V_o = 5$V and a 100mA load current:

$$P_d = (V_i - 5) \cdot 0.1 \leq 1.49$$

from which we solve $V_i \leq 19.9$V.

The moral: there are multiple limits within which any electronic device must operate. Yes, the 7805 datasheet says you must not apply more than 35V to the 7805, but that is in the general case. The junction temperature limit places an additional constraint on the maximum input voltage.

Laboratory Part V – Paralleling Linear Regulators

The current to the load should have been shared between the regulators, though not necessarily equally. Section 1.10 explains why.

Laboratory Part VI – Linear Regulator Transient Response

You should have observed an effect similar to Figure 27 on page 27 of your lecture notes (with a lot of extra "noise" thrown in not revealed by simulation).