

7. KARNAUGH MAPS

Topics:

- Truth tables and Karnaugh maps

Objectives:

- Be able to simplify designs with Boolean algebra and Karnaugh maps

7.1 INTRODUCTION

Karnaugh maps allow us to convert a truth table to a simplified Boolean expression without using Boolean Algebra. The truth table in Figure 85 is an extension of the previous burglar alarm example, an alarm quiet input has been added.

Given

A, W, M, S as before
 Q = Alarm Quiet (0 = quiet)

Step1: Draw the truth table

S	M	W	Q	A
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

Figure 85 Truth Table for a Burglar Alarm

Instead of converting this directly to a Boolean equation, it is put into a tabular form as shown in

Figure 86. The rows and columns are chosen from the input variables. The decision of which variables to use for rows or columns can be arbitrary - the table will look different, but you will still get a similar solution. For both the rows and columns the variables are ordered to show the values of the bits using NOTs. The sequence is not binary, but it is organized so that only one of the bits changes at a time, so the sequence of bits is 00, 01, 11, 10 - this step is very important. Next the values from the truth table that are true are entered into the Karnaugh map. Zeros can also be entered, but are not necessary. In the example the three true values from the truth table have been entered in the table.

Step 2: Divide the input variables up. I choose SQ and MW

Step 3: Draw a Karnaugh map based on the input variables

	$\overline{M} \overline{W} (=00)$	$\overline{M} W (=01)$	$M W (=11)$	$M \overline{W} (=10)$
$\overline{S} \overline{Q} (=00)$				
$\overline{S} Q (=01)$				
$S Q (=11)$	1		1	1
$S \overline{Q} (=10)$				

Added for clarity

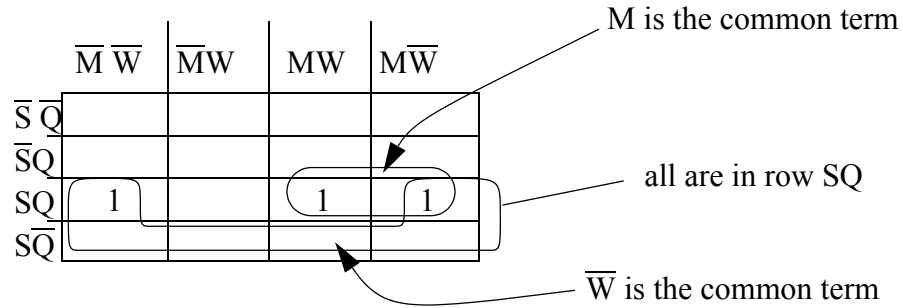
Note: The inputs are arranged so that only one bit changes at a time for the Karnaugh map. In the example above notice that any adjacent location, even the top/bottom and left/right extremes follow this rule. This is done so that changes are visually grouped. If this pattern is not used then it is much more difficult to group the bits.

Figure 86 The Karnaugh Map

When bits have been entered into the Karnaugh map there should be some obvious patterns. These patterns typically have some sort of symmetry. In Figure 87 there are two patterns that have been circled. In this case one of the patterns is because there are two bits beside each other. The second pattern is harder to see because the bits in the left and right hand side columns are beside each other. (Note: Even though the table has a left and right hand column, the sides and top/bottom wrap around.) Some of the bits are used more than once, this will lead to some redundancy in the final equation, but it will also give a simpler expression.

The patterns can then be converted into a Boolean equation. This is done by first observing that all of the patterns sit in the third row, therefore the expression will be ANDed with SQ. There are two patterns in the third row, one has M as the common term, the second has \overline{W} as the common term. These can now be combined into the equation. Finally the equation is converted to ladder logic.

Step 4: Look for patterns in the map



Step 5: Write the equation using the patterns

$$A = S \cdot Q \cdot (M + \bar{W})$$

Step 6: Convert the equation into ladder logic

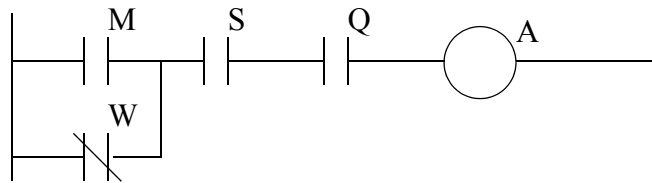


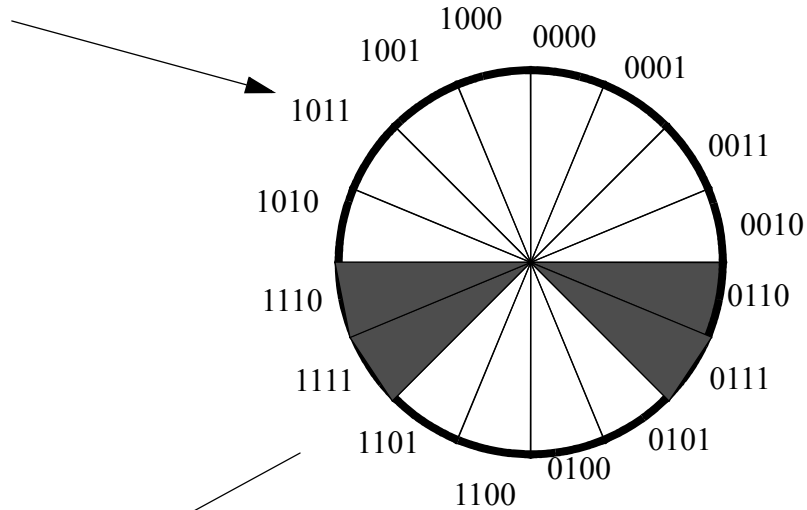
Figure 87 Recognition of the Boolean Equation from the Karnaugh Map

Karnaugh maps are an alternative method to simplifying equations with Boolean algebra. It is well suited to visual learners, and is an excellent way to verify Boolean algebra calculations. The example shown was for four variables, thus giving two variables for the rows and two variables for the columns. More variables can also be used. If there were five input variables there could be three variables used for the rows or columns with the pattern 000, 001, 011, 010, 110, 111, 101, 100. If there is more than one output, a Karnaugh map is needed for each output.

Aside: A method developed by David Luque Sacaluga uses a circular format for the table. A brief example is shown below for comparison.

A	B	C	D	X
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

Convert the truth table to a circle using the Gray code for sequence. Bits that are true in the truth table are shaded in the circle.



Look for large groups of repeated patterns.

1. In this case 'B' is true in the bottom half of the circle, so the equation becomes,

$$X = B \cdot (\dots)$$

2. There is left-right symmetry, with 'C' as the common term, so the equation becomes

$$X = B \cdot C \cdot (\dots)$$

3. The equation covers all four values, so the final equation is,

$$X = B \cdot C$$

Figure 88 Aside: An Alternate Approach

7.2 SUMMARY

- Karnaugh maps can be used to convert a truth table to a simplified Boolean equation.

7.3 PRACTICE PROBLEMS

1. Setup the Karnaugh map for the truth table below.

A	B	C	D	Result
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

2. Use a Karnaugh map to simplify the following truth table, and implement it in ladder logic.

A	B	C	D	X
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

3. Write the simplest Boolean equation for the Karnaugh map below,

	CD	$C\bar{D}$	$\bar{C}\bar{D}$	$\bar{C}D$
AB	1	0	0	1
$A\bar{B}$	0	0	0	0
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	1	1	0

4. Given the truth table below find the most efficient ladder logic to implement it. Use a structured technique such as Boolean algebra or Karnaugh maps.

A	B	C	D	X	Y
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	0	0
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	1	1
1	0	1	0	0	0
1	0	1	1	0	0
1	1	0	0	1	0
1	1	0	1	1	0
1	1	1	0	0	1
1	1	1	1	0	1

5. Examine the truth table below and design the simplest ladder logic using a Karnaugh map.

D	E	F	G	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

6. Find the simplest Boolean equation for the Karnaugh map below without using Boolean algebra to simplify it. Draw the ladder logic.

	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}B\bar{C}$	$\bar{A}BC$	$A\bar{B}\bar{C}$	$A\bar{B}C$	$AB\bar{C}$	ABC
$\bar{D}\bar{E}$	1	1	0	1	0	0	0	0
$\bar{D}E$	1	1	0	0	0	0	0	0
DE	1	1	0	0	0	0	0	0
$D\bar{E}$	1	1	0	1	0	0	0	0

7. Given the following truth table for inputs A, B, C and D and output X. Convert it to simplified ladder logic

using a Karnaugh map.

A	B	C	D	X
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

8. Consider the following truth table. Convert it to a Karnaugh map and develop a simplified Boolean equation

(without Boolean algebra). Draw the corresponding ladder logic.

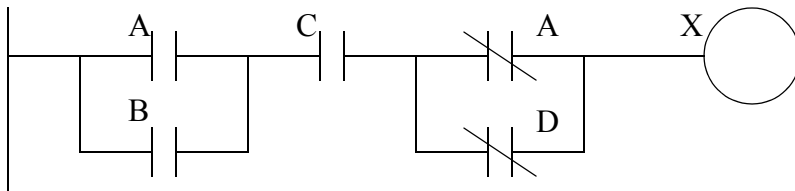
inputs					output
A	B	C	D	E	X
0	0	0	0	0	
0	0	0	0	1	
0	0	0	1	0	
0	0	0	1	1	
0	0	1	0	0	
0	0	1	0	1	
0	0	1	1	0	1
0	0	1	1	1	1
0	1	0	0	0	
0	1	0	0	1	
0	1	0	1	0	1
0	1	0	1	1	1
0	1	1	0	0	
0	1	1	0	1	
0	1	1	1	0	
0	1	1	1	1	1
1	0	0	0	0	
1	0	0	0	1	
1	0	0	1	0	
1	0	0	1	1	
1	0	1	0	0	
1	0	1	0	1	
1	0	1	1	0	
1	0	1	1	1	1
1	1	0	0	0	
1	1	0	0	1	
1	1	0	1	0	
1	1	0	1	1	
1	1	1	0	0	1
1	1	1	0	1	1
1	1	1	1	0	1
1	1	1	1	1	1

9. Given the truth table below

A	B	C	D	Z
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

- a) find a Boolean algebra expression using a Karnaugh map.
 b) draw a ladder diagram using the truth table (not the Boolean expression).

10. Convert the following ladder logic to a Karnaugh map.



11. a) Construct a truth table for the following problem.
 i) there are three buttons A, B, C.
 ii) the output is on if any two buttons are pushed.
 iii) if C is pressed the output will always turn on.
 b) Develop a Boolean expression.
 c) Develop a Boolean expression using a Karnaugh map.

12. Develop the simplest Boolean expression for the Karnaugh map below,
 a) graphically.
 b) by Boolean Algebra

	$\bar{A}\bar{B}$	$A\bar{B}$	AB	$\bar{A}B$
CD	1			1
$C\bar{D}$		1	1	
$\bar{C}\bar{D}$				
$\bar{C}D$	1			1

13. Consider the following boolean equation.

$$X = \overline{(A + B\bar{A})\bar{A}} + \overline{(CD + \bar{C}D + C\bar{D})}$$

- a) Can this Boolean equation be converted directly ladder logic. Explain your answer, and if necessary, make any changes required so that it may be converted to ladder logic.

- b) Write out ladder logic, based on the result in step a).
- c) Simplify the equation using Boolean algebra and write out new ladder logic.
- d) Write a Karnaugh map for the Boolean equation, and show how it can be used to obtain a simplified Boolean equation.

7.4 ASSIGNMENT PROBLEMS

1. Use the Karnaugh map below to create a simplified Boolean equation. Then use the equation to create ladder logic.

	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	1	1
$C\bar{D}$	1	0	0	1
$\bar{C}D$	0	0	0	1
$\bar{C}\bar{D}$	0	0	0	1

2. Use a Karnaugh map to develop simplified ladder logic for the following truth table where A, B, C and D are inputs, and X and Y are outputs.

A	B	C	D	X	Y
0	0	0	0	0	0
0	0	0	1	1	0
0	0	1	0	0	0
0	0	1	1	1	0
0	1	0	0	0	0
0	1	0	1	0	1
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	0	0
1	0	0	1	1	0
1	0	1	0	0	0
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	1	1
1	1	1	0	0	1
1	1	1	1	1	1

3. You are planning the basic layout for a control system with the criteria provided below. You need to plan the wiring for the input and output cards, and then write the ladder logic for the controller. You decide to use a Boolean logic design technique to design the ladder logic. AND, your design will be laid out on the design sheets found later in this book.

- There are two inputs from PNP photoelectric sensors *part* and *busy*.

- There is a NO *cycle* button, and NC *stop* button.
- There are two outputs to indicator lights, the *running* light and the *stopped* light.
- There is an output to a conveyor, that will drive a high current 120Vac motor.
- The conveyor is to run when the *part* sensor is on and while the *cycle* button is pushed, but the *busy* sensor is off. If the *stop* button is pushed the conveyor will stop.
- While the conveyor is running the *running* light will be on, otherwise the *stopped* light will be on.

4. Convert the following truth table to simplified ladder logic using a Karnaugh map AND Boolean equations. The inputs are A, B, C and D and the output is X.

A	B	C	D	X
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0