25. THERMAL SYSTEMS

Topics:

Objectives:

25.1 INTRODUCTION

• Energy can be stored and transferred in materials in a number of forms. Thermal energy (heat) is stored and transmitted through all forms of matter.

25.2 MATHEMATICAL PROPERTIES

25.2.1 Resistance

• A universal property of energy is that it constantly strives for equilibrium. This means that a concentration of energy will tend to dissipate. When material separates regions of different temperatures it will conduct heat energy at a rate proportional to the difference. Materials have a measurable conductivity or resistance (note: this is different than electrical conductivity and resistance.)

$$q = \frac{1}{R}(\theta_j - \theta_i) \qquad\qquad \frac{dq}{dl} = \frac{1}{R}\left(\frac{d\theta}{dl}\right)$$

where,

q = heat flow rate from j to i (J/s or watts)

R = thermal resistance

θ = temperature

$$R = \frac{d}{A\alpha}$$

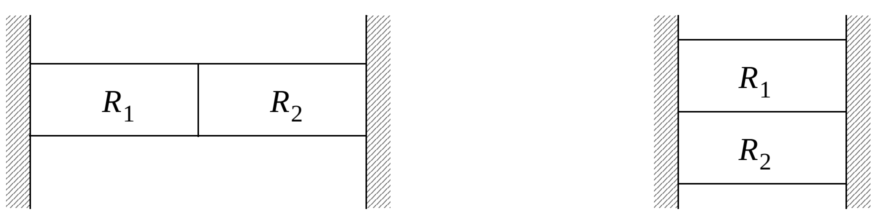
d = length of thermal conductor

A = cross section area of thermal conductor

α = thermal conductivity (W/mK)

Figure 25.1 Thermal resistance

• When dealing with thermal resistance there are many parallels to electrical resistance. The flow of heat (current) is proportional to the thermal difference (voltage). If we have thermal systems in parallel or series they add as normal resistors do.



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \qquad\qquad R_T = R_1 + R_2$$

Figure 25.2 Parallel and series thermal resistance

25.2.2 Capacitance

• Heat energy is absorbed at different rates in different materials - this rate is referred to as thermal capacitance. And as long as the material has no major changes in structure (i.e., gas-solid or phase transition) this number is relatively constant.

$$\Delta\theta = \frac{1}{C}(q_{in} - q_{out}) \qquad \frac{d\theta}{dt} = \frac{1}{C}\frac{dq}{dt}$$

where,

C = thermal capacitance

$$C = M\sigma$$

where,

M = mass of thermal body
σ = specific heat of material in mass

Figure 25.3 Thermal capacitance

• One consideration when dealing with heating capacitance is that the heat will not instantly disperse throughout the mass. When we want to increase the rate of heat absorption we can use a mixer (with a gas or fluid). A mixer is shown in the figure below. This mixer is just a rotating propeller that will cause the liquid to circulate.

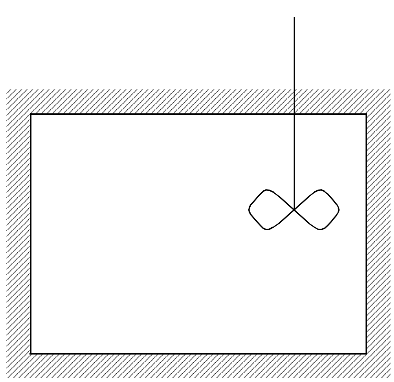


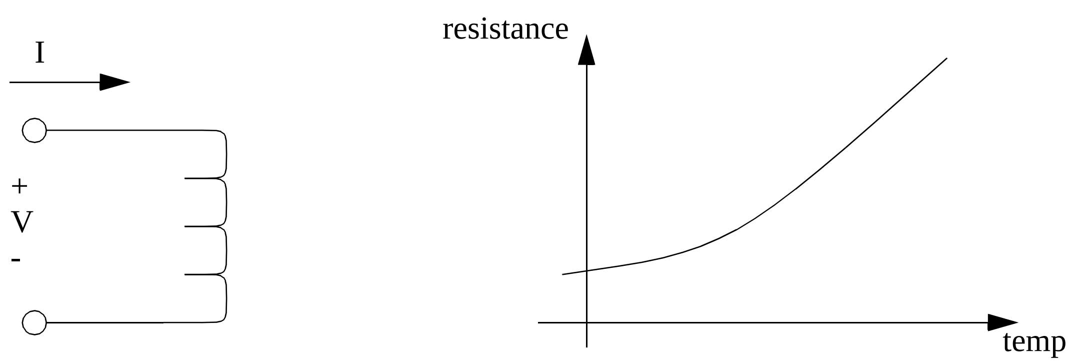
Figure 25.4 A mixer to prevent uneven temperature distribution in thermal storage

25.2.3 Sources

• If we plan to design a thermal system we will need some sort of heat source. One popular heating source is an electrical heating element.

• heating coils are normally made out of some high resistance metal/alloy such as nichrome (nickel and chrome) or tungsten.

• If we run a current through the wire the resistance will generate heat. As these metals get hotter the resistance rises, hence the temperature is self regulating. If we control the voltage and current we can control the amount of heat delivered by the coil.



$$P = IV$$

where,

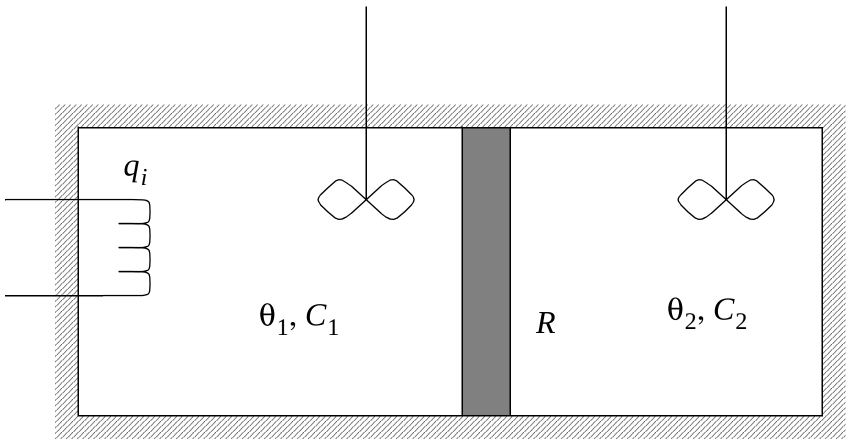
P = power generated as heat
I = current into coil
V = voltage across coil

Figure 25.5 Electrical heating elements

25.3 EXAMPLE SYSTEMS

• We know that as we heat materials at one point, they do not instantly heat at all points, there is some delay (consider a spoon in a hot bowl of soup). This delay is a function of heat capacitance (yes there is a parallel to the electrical description).

• Consider an insulated sealed chamber with a resistive barrier between sides, and a heating element in one side.



Given

q_0 = heat flow into the left chamber from the heating element

θ_1, C_1 = initial temperature and heat capacitance of left side

θ_2, C_2 = initial temperature and heat capacitance of right side

Next,

q_1 = heat flow through the center barrier

$$q_1 = \frac{1}{R}(\theta_1 - \theta_2)$$

$$\frac{d}{dt}\theta_1 = \frac{1}{C_1}(q_0 - q_1) \qquad \frac{d}{dt}\theta_2 = \frac{1}{C_2}(q_1)$$

$$s\theta_1 = \frac{1}{C_1}\left(q_0 - \frac{1}{R}(\theta_1 - \theta_2)\right) \qquad s\theta_2 = \frac{1}{RC_2}(\theta_1 - \theta_2)$$

$$Rq_o - sRC_1\theta_1 = \theta_1 - \theta_2 \qquad \theta_1 = \theta_2 + sRC_2\theta_2$$

$$\theta_1(1 + sRC_1) = \theta_2 + Rq_o$$

$$\theta_1 = \frac{\theta_2 + Rq_o}{1 + sRC_1}$$

Figure 25.6 A two chamber thermal system

$$\theta_1 = \frac{\theta_2 + Rq_o}{1 + sRC_1} = \theta_2 + sRC_2\theta_2$$

$$\frac{\theta_2 + Rq_o}{\theta_2 + sRC_2\theta_2} = 1 + sRC_1$$

$$\frac{1}{1 + sRC_2} + \frac{\left(\frac{Rq_o}{\theta_2}\right)}{1 + sRC_2} = 1 + sRC_1$$

$$1 + \left(\frac{Rq_o}{\theta_2}\right) = 1 + s(RC_1 + RC_2) + s^2R^2C_1C_2$$

$$\frac{Rq_o}{\theta_2} = s(RC_1 + RC_2) + s^2R^2C_1C_2$$

$$\frac{q_o}{\theta_2} = s(C_1 + C_2) + s^2RC_1C_2$$

$$\theta_2 = \frac{\left(\frac{q_o}{RC_1C_2}\right)}{s\left(\frac{C_1 + C_2}{RC_1C_2}\right) + s^2} = \frac{A}{s} + \frac{B}{s + \frac{C_1 + C_2}{RC_1C_2}}$$

$$A = \lim_{s \to 0}\left(\frac{\left(\frac{q_o}{RC_1C_2}\right)}{s\left(\frac{C_1 + C_2}{RC_1C_2}\right) + s^2}s\right) = \frac{\left(\frac{q_o}{RC_1C_2}\right)}{\left(\frac{C_1 + C_2}{RC_1C_2}\right) + (0)} = \frac{q_o}{C_1 + C_2}$$

$$B = \lim_{s \to -\frac{C_1 + C_2}{RC_1C_2}}\left(\frac{\left(\frac{q_o}{RC_1C_2}\right)}{s\left(\frac{C_1 + C_2}{RC_1C_2}\right) + s^2}\left(s + \frac{C_1 + C_2}{RC_1C_2}\right)\right) = \frac{\left(\frac{q_o}{RC_1C_2}\right)}{-\left(\frac{C_1 + C_2}{RC_1C_2}\right)} = \frac{-q_o}{C_1 + C_2}$$

$$\theta_2 = \frac{\frac{q_o}{C_1 + C_2}}{s} + \frac{\frac{-q_o}{C_1 + C_2}}{s + \frac{C_1 + C_2}{RC_1C_2}}$$

Figure 25.7 A two chamber thermal system (continued)

$$L^{-1}(\theta_2) = \frac{q_o}{C_1 + C_2}\left[L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{s + \frac{C_1 + C_2}{RC_1C_2}}\right]\right]$$

$$\theta_2(t) = \frac{q_o}{C_1 + C_2}\left[1 - e^{-\left(\frac{C_1 + C_2}{RC_1C_2}\right)t}\right]$$

Figure 25.8 A two chamber thermal system (continued)

25.4 SUMMARY

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25.5 PRACTICE PROBLEMS

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25.6 PRACTICE PROBLEM SOLUTIONS

25.7 ASSIGNMENT PROBLEMS