# 12. NONLINEAR SYSTEMS

Topics:

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Objectives:

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## 12.1 INTRODUCTION

• how they are different from linear

- no transfer functions

- new elements needed in block diagrams

## 12.2 SOURCES OF NONLINEARITY

- important for analysis

- can be overcome by control strategies

### 12.2.1 Non-Linear Relationships

- input/output relationships are not linear

$$Q(\theta) = 8.5\left(e^{-\frac{\theta}{\pi}} - 1\right)$$

## 12.3 NON-LINEAR ELEMENTS

• If our models include a device that is non linear we will need to linearize the model before we can proceed.

• A non-linear system can be approximated with a linear equation using the following method.

1. Pick an operating point or range for the component.
2. Find a constant value that relates a change in the input to a change in the output.
3. Develop a linear equation.
4. Use the linear equation in the analysis (Laplace or other)

• Consider the example below,

In this case the relationship between pressure drop and flow are non-linear. We need to develop an equation that approximates the local operating point.

$$\Delta p \approx R(q - \bar{q})$$

$$R \approx \frac{\Delta(\Delta p)}{\Delta q} \qquad R = 2\frac{q}{K^2}$$

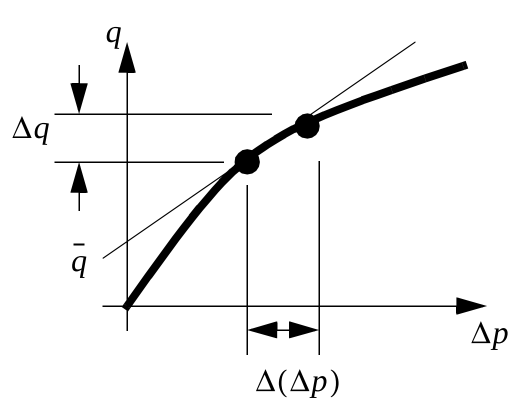

*Figure 12.1* Linearizing non-linear elements

## 12.3.1 Time Variant

- system parameters vary as a function of time.

$$\ddot{x} + \dot{x}\left(\frac{K_d}{M - 0.0001t}\right) + x\left(\frac{K_s}{M - 0.0001t}\right) = \frac{F}{M - 0.0001t}$$

## 12.3.2 Switching

- system components turned on/off

- cables in tension/compression

- show an example where input conditions change

$$\dot{y} + 4y = f(t) \qquad f(t < 0) = 0$$

$$f(0 \le t < 5s) = 2$$

$$f(t \ge 5s) = 0$$

- give PWM (Pulse Width Modulation) example with ripple showing equivalent voltage. PWM is used to generate analog voltage equivalents. Show for a system with first order response with tau = 0.1s for a frequency of 1KHz, 10Hz and 1Hz. Point out the ripple and effective voltage.

- important to consider when doing system analysis

### 12.3.3 Deadband

- Friction in all components

- costs money to reduce friction, so it is better to compensate in software

- small actuation signals not large enough to overcome friction

- This effect is normally known as ’stiction’, a combination of the words static and friction.

- Friction is common in less expensive motors, and when a motor is driving a mechanical system.

- In systems there are two type of friction that must be considered.

- The static friction, ’stiction’, will prevent initial motion. If the systems breaks free and starts turning, the kinetic friction will provide a roughly constant friction resistance.

- relationship in figure below.

- the region where the applied voltage has no effect is called the deadband.

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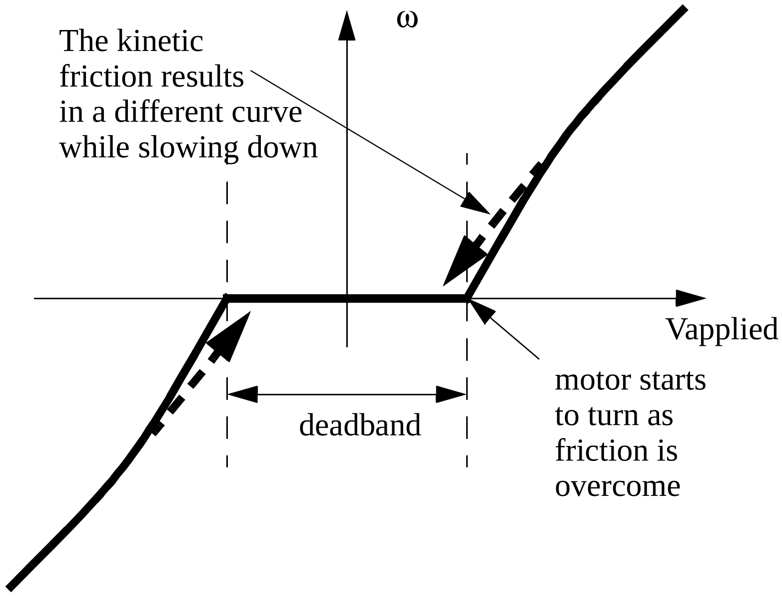



*Figure 12.2* Motor deadband for a bidirectional motor

- deadband compensation as shown in figure below.

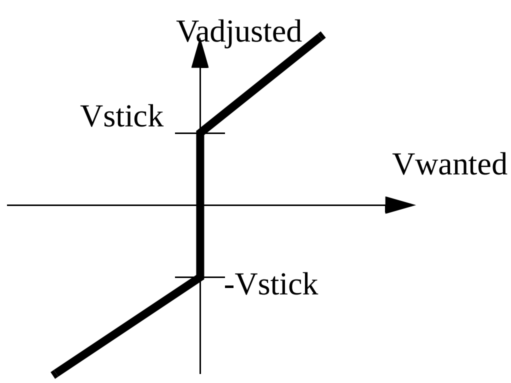
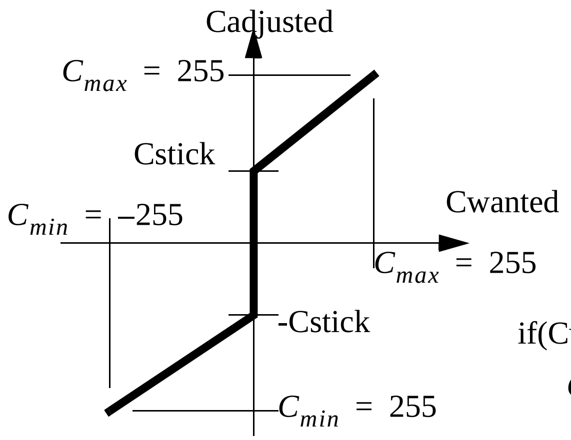



*Figure 12.3* Deadband approximation for a bidirectional motor

- equations for these are shown in figure




if(Cwanted > 0)

$$C_{adjusted} = C_{stick} + \left(\frac{C_{wanted}}{C_{max}}\right)(C_{max} - C_{stick})$$

if(Cwanted < 0)

$$C_{adjusted} = -C_{stick} + \left(\frac{C_{wanted}}{C_{min}}\right)(C_{min} - C_{stick})$$

if(Cwanted = 0)

$$C_{adjusted} = 0$$

*Figure 12.4* Deadband approximation for a bidirectional motor

- c-code below

```
#define c_stick_pos    100
#define c_stick_neg    -110 /* make the value positive */
#define c_max          255
#define c_min          -255 /* make the value positive */

int deadband(int c_wanted){ /* call this routine when updating */
        int     c_adjusted;

        if(c_wanted == 0){ /* turn off the output */
                c_adjusted = 0;
        } else if(c_wanted > 0){ /* a positive output */
                c_adjusted = c_stick_pos +
                        (c_max - c_stick_pos) * c_wanted / c_max;
                if(c_adjusted > c_max) c_adjusted = c_max;
        } else { /* the output must be negative */
                c_adjusted = -c_stick_neg -
                        (c_min - c_stick_neg) * c_wanted / c_min;
                if(c_adjusted < -c_min) c_adjusted = c_min;
        }

        return c_adjusted;

}
```

*Figure 12.5* Deadband compensation subroutine

## 12.3.4 Saturation and Clipping

• Some devices have natural maximum values, such as voltage or pressure limitations caused by a regulated supply.

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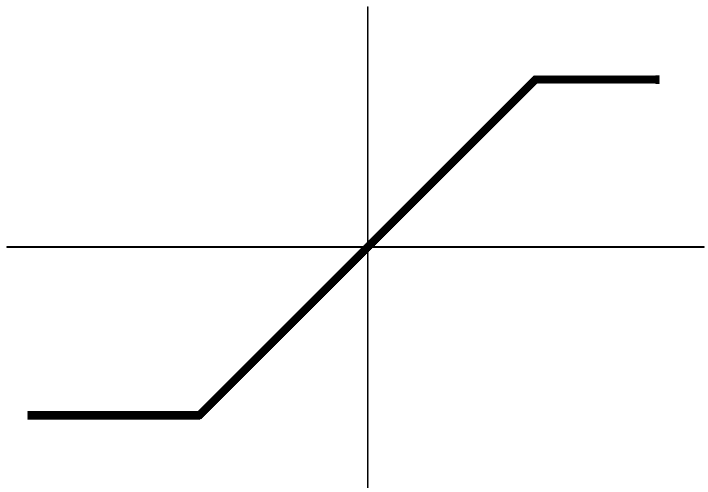

## 12.3.5 Hysteresis and Slip

- windup resulting from springiness and friction

- backlash

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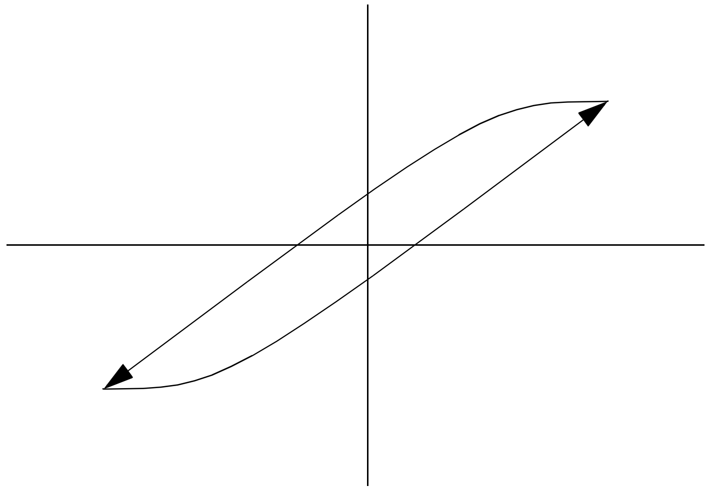

- correct by tracking the previous motion direction and taking extra steps when reversing direction

## 12.3.6 Delays and Lags

• Time delays are common in systems

• In the simplest form this is a period of time between when an event occurs and when the effect occurs.

• If an output delay is larger than the control system step time it may be necessary to predict future states and initiate outputs ahead of those.

• If an input delay is larger than the control system it might be necessary to slow the control action, or build it into the control law.

## 12.4 SUMMARY

## 12.5 PRACTICE PROBLEMS

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## 12.6 PRACTICE PROBLEM SOLUTIONS

## 12.7 ASIGNMENT PROBLEMS