23. ELECTROMECHANICAL SYSTEMS

Topics:

Objectives:

23.1 INTRODUCTION

- Magnetic fields and forces are extremely useful. The fields can allow energy storage, or transmit forces.

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23.2 MATHEMATICAL PROPERTIES

- Magnetic fields have direction. As a result we must pay special attention to directions, and vector calculations.

23.2.1 Induction

- Magnetic fields pass through space.

- resistivity of materials decreases with temperature

- Amperes Circuit Law

$$I = \oint H dl$$

where,

I = current flowing along a line (A)

H = magnetic field intensity (A/m)

l = A perpendicular path around the current flow (m)

For example, at a fixed radius (r) around a wire,

$$I = \int_0^{2\pi} H r d\theta$$

$$\therefore I = 2\pi H r$$

$$\therefore H = \frac{I}{2\pi r}$$

- Flux density can be calculated for low H values. As the value climbs the relationship becomes non-linear.

$$B = \mu H = \mu_r \mu_0 H$$

where,

B = Flux density (Wb/m*2 or T)

μ = permeability of material

μ_0 = permeability of free space = $4\pi\times 10^{-7}\frac{Henry}{m}$

μ_r = relative permeability

- Permeability,

$$\mu = \frac{B}{H}$$

$$\mu_0 = 4\pi 10^{-7}\frac{H}{m}$$

$$\mu = \mu_r \mu_0$$

where,

H = Magnetic field intensity $\left(\frac{A}{m}\right)$
μ_0 = permeability of free space
μ_r = relative permeability of a material
μ = permeability of a material

• Permeability is approximately linear for smaller electric fields, but with larger magnetic fields the materials saturate and the value of B reaches a maximum value.

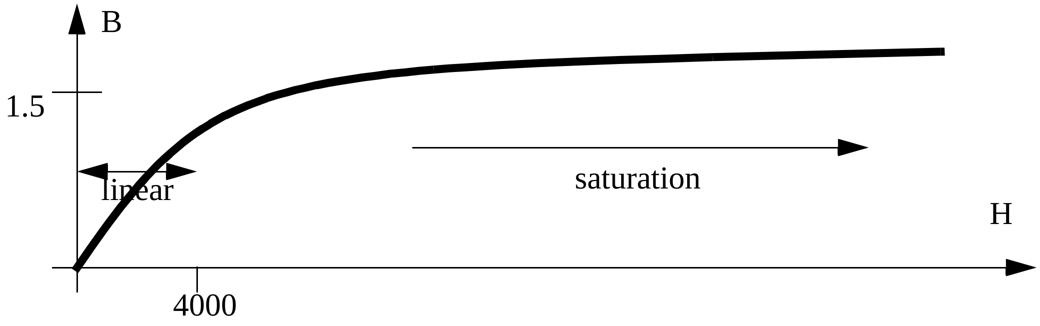


Figure 23.1 Saturation for a mild steel (approximately)

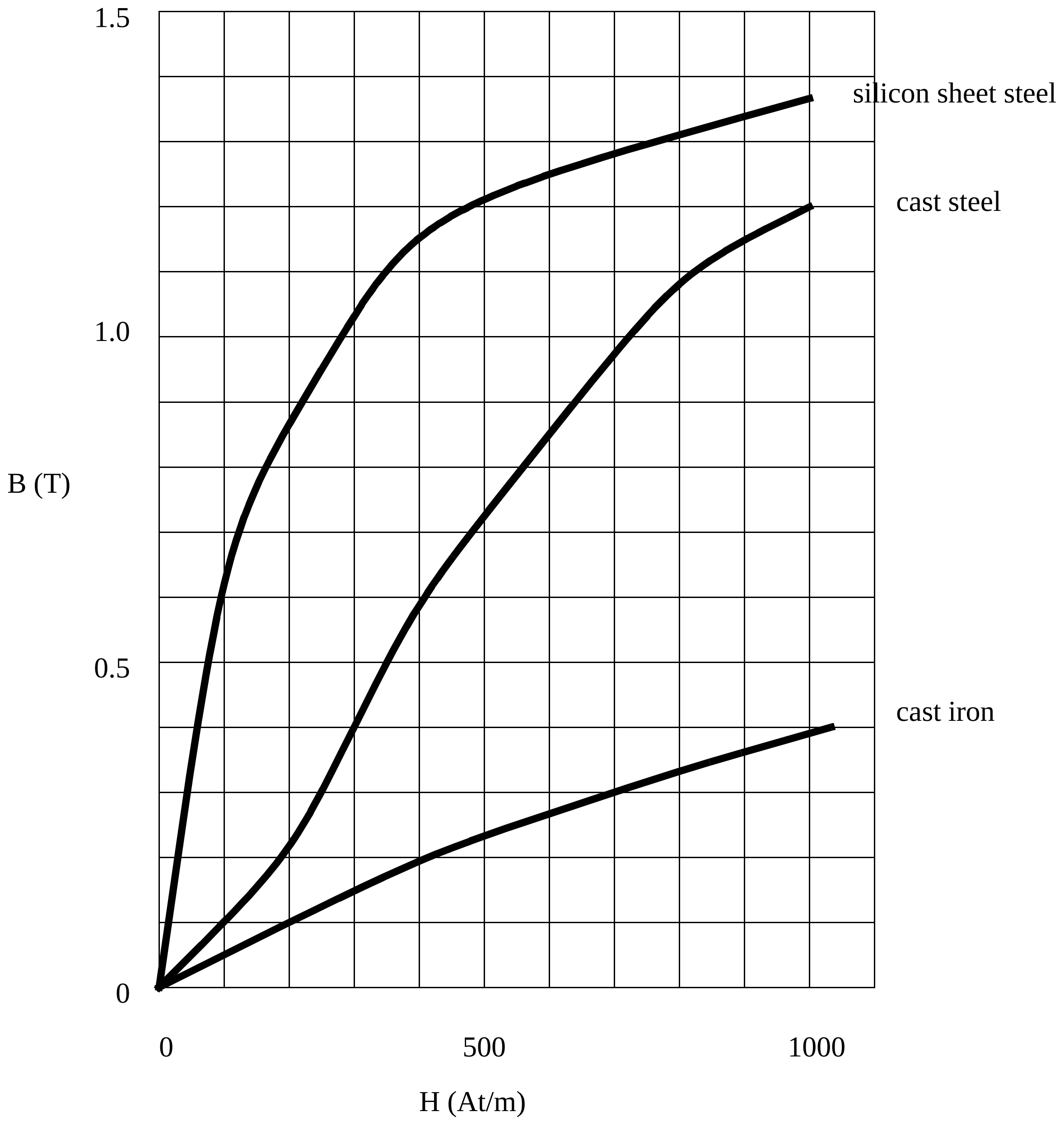


Figure 23.2 Magnetization curves (Sen, 1989)

- Flux density about a wire

$$B = \frac{I}{2\pi r}$$ For an infinitely long straight conductor

where,

B = Flux density$\left(\frac{Wb}{m^2} \text{ or } Tesla\right)$

I = Current in the conductor (A)

r = radial distance from the conductor

- Flux and flux density,

$$\Phi = \int BdA$$

where,

Φ = Flux density (Wb)

A = Cross section area perpendicular to flux

- When a material is used out of the saturation region the permeabilities may be written as reluctances,

$$R = \frac{L}{\mu A}$$

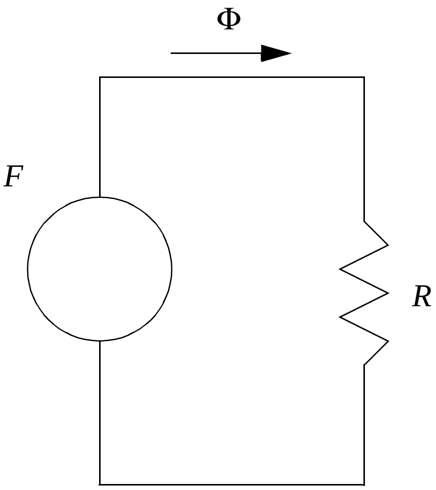
where,

R = reluctance of a magnetic path

L = length of a magnetic path

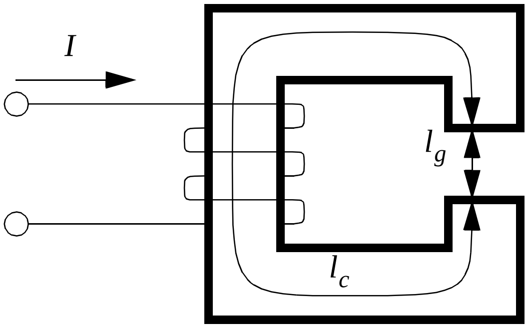
A = cross section area of a magnetic path

- Electric circuit analogy



$$\Phi = \frac{Ni}{\left(\frac{l}{\mu A}\right)} = \frac{F}{R}$$

- Example,



$$R_c = \frac{l_c}{\mu_c A_c}$$

$$R_g = \frac{l_g}{\mu_g A_g}$$

$$\Phi = \frac{Ni}{R_c + R_g}$$

- Faraday’s law,

$$e = N\frac{d}{dt}\Phi$$ For a coil

where,

e = the potential voltage across the coil

N = the number of turns in the coil

- Field energy,

$$W = \frac{B^2}{2\mu}V = \frac{B^2}{2\mu}Al$$

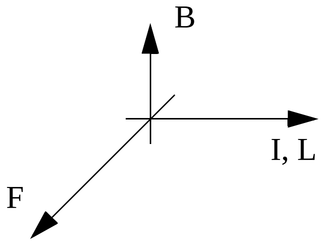
- Force can be derived from the energy,

$$F = \frac{B^2}{2\mu}A$$

- The basic property of induction is that it will (in the presence of a magnetic field) convert a changing current flowing in a conductor to a force or convert a force to a current flow from a change in the current or the path.

$$F = (I \times B)L$$

$$F = (L \times B)I$$



NOTE: As with all cross products we can use the right hand rule here.

where,

L = conductor length

F = force (N)

The FBD/schematic equivalent is,

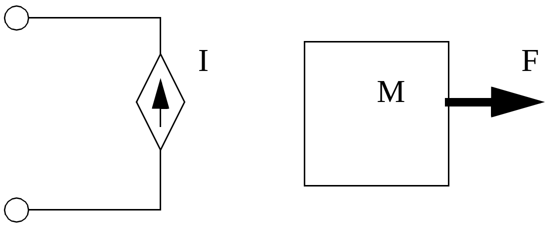


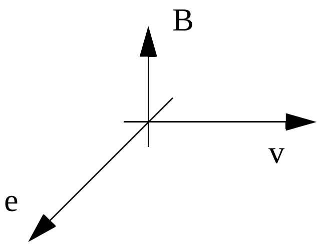
Figure 23.3 The current and force relationship

• We will also experience an induced current caused by a conductor moving in a magnetic field. This is also called emf (Electro-Motive Force)

$$e_m = (v \times B)L$$

where,

e_m = electromotive force (V)
ϕ = magnetic flux (Wb - webers)
v = velocity of conductor



The FBD/schematic equivalent

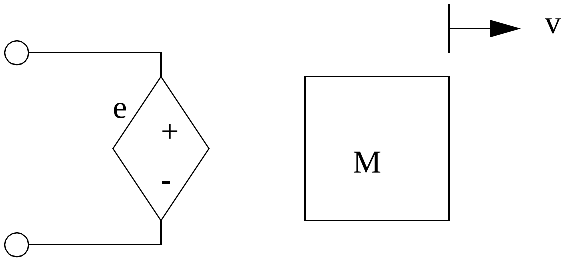


Figure 23.4 Electromagnetically induced voltage

- Hysteresis

23.3 EXAMPLE SYSTEMS

- These systems are very common, take for example a DC motor. The simplest motor has a square conductor loop rotating in a magnetic field. By applying voltage the wires push back against the magnetic field.

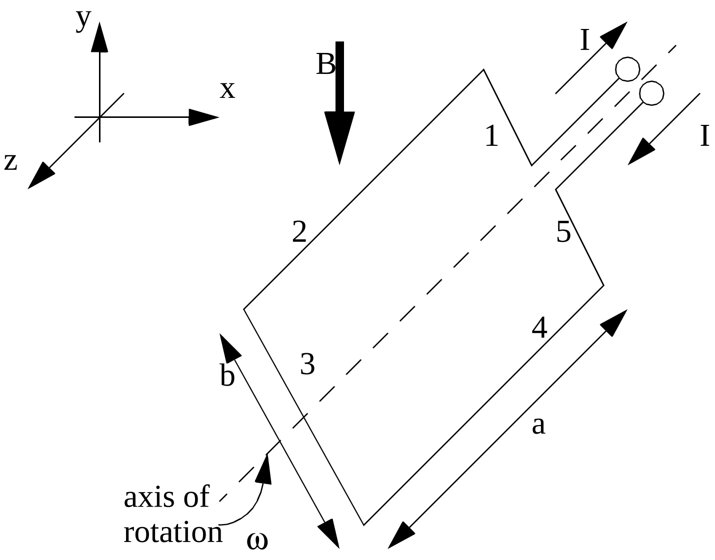


Figure 23.5 A motor winding in a magnetic field

For wire 3,

$$P_3 = \begin{bmatrix} r\cos(\omega t) \\ r\sin(\omega t) \\ -\frac{b}{2}\text{to}\frac{b}{2} \end{bmatrix} \qquad V_3 = \begin{bmatrix} -r\omega\sin(\omega t) \\ r\omega\cos(\omega t) \\ 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ -B \\ 0 \end{bmatrix}$$

$$de_{m3} = (V \times B)dL$$

$$e_{m3} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \begin{bmatrix} -r\omega\sin(\omega t) \\ r\omega\cos(\omega t) \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ -B \\ 0 \end{bmatrix} dr$$

$$e_{m3} = \int_{-\frac{b}{2}}^{\frac{b}{2}} \begin{bmatrix} 0 \\ 0 \\ Br\omega\sin(\omega t) \end{bmatrix} dr$$

$$e_{m3} = B\frac{r^2}{2}\omega\sin(\omega t)\Bigg|_{-\frac{b}{2}}^{\frac{b}{2}} = B\omega\sin(\omega t)\left[\left(\frac{b}{2}\right) - \left(-\frac{b}{2}\right)^2\right] = 0$$

For wires 1 and 5,

By symmetry, the two wires together will act like wire 3. Therefore they both have an emf (voltage) of 0V.

$$e_{m1} = e_{m5} = 0V$$

Figure 23.6 Calculation of the motor torque

For wire 2 (and 4 by symmetry),

$$P_2 = \begin{bmatrix} \frac{b}{2}\cos(\omega t) \\ \frac{b}{2}\sin(\omega t) \\ 0 \text{to} a \end{bmatrix} \qquad V_2 = \begin{bmatrix} -\frac{b}{2}\omega\sin(\omega t) \\ \frac{b}{2}\omega\cos(\omega t) \\ 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ -B \\ 0 \end{bmatrix}$$

$$de_{m2} = (V \times B)dL$$

$$e_{m2} = \int_0^a \begin{bmatrix} -\frac{b}{2}\omega\sin(\omega t) \\ \frac{b}{2}\omega\cos(\omega t) \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ -B \\ 0 \end{bmatrix} dl$$

$$e_{m2} = \int_0^a \begin{bmatrix} 0 \\ 0 \\ B\frac{b}{2}\omega\cos(\omega t) \end{bmatrix} dl = aB\frac{b}{2}\cos(\omega t)$$

$$e_{m4} = e_{m2}$$

For the total loop,

$$e_m = e_{m1} + e_{m2} + e_{m3} + e_{m4} + e_{m5}$$

$$e_m = 0 + aB\frac{b}{2}\omega\cos(\omega t) + 0 + aB\frac{b}{2}\omega\cos(\omega t) + 0$$

$$e_m = aBb\omega\cos(\omega t)$$

Figure 23.7 Calculation of the motor torque (continued)

- As can be seen in the previous equation, as the loop is rotated a voltage will be generated (a generator), or a given voltage will cause the loop to rotate (motor).

- In this arrangement we have to change the polarity on the coil every 180 deg of rotation. If we didn't do this the loop the torque on the loop would reverse for half the motion. The result would be that the motor would swing back and forth, but not rotate fully. To make the torque push consistently in the same direction we need to reverse the applied voltage for half the cycle. The device that does this is called a commutator. It is basically a split ring with brushes.

$$e_m = aBb\omega|\cos(\omega t)|$$

• Real motors also have more than a single winding (loop of wire). To add this into the equation we only need to multiply by the number of loops in the winding.

$$e_m = NaBb\omega|\cos(\omega t)|$$

• As with most devices the motor is coupled. This means that one change, say in torque/force will change the velocity and hence the voltage. But a change in voltage will also change the current in the windings, and hence the force, etc.

• Consider a motor that is braked with a constant friction load of Tf.

$$F_w = (I \times B)L = IBa$$

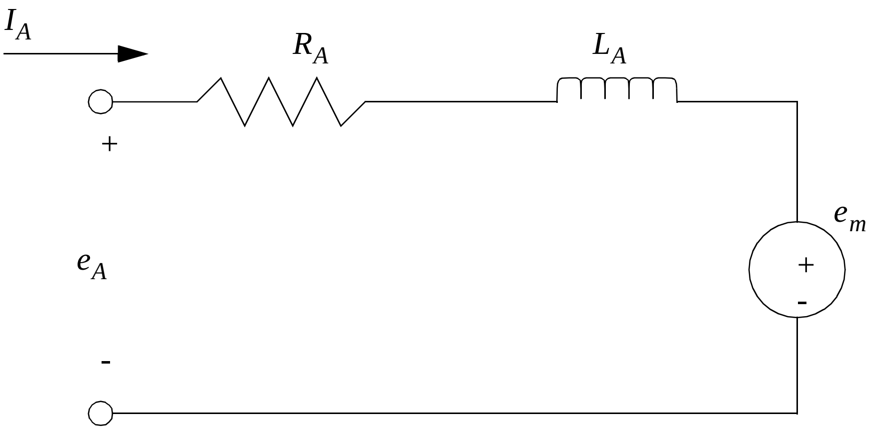
$$T_w = 2\left(F \times \frac{b}{2}\right) = Fb = IBab$$

$$\sum M = T_w - T_f = J\alpha$$

$$IBab - T_f = J\alpha$$

Figure 23.8 Calculation of the motor torque (continued)

• We still need to relate the voltage and current on the motor. The equivalent circuit for a motor shows the related components.



where,

I_A, e_A = voltage and current applied to the armature (motor supply)
R_A, L_A = equivalent resistance and inductance of windings

$$\sum V = e_A - I_A R_A - L_A \frac{d}{dt} I_A - e_m = 0$$

we can now add in the other equations,

$$e_A - I_A R_A - L_A \frac{d}{dt} I_A - NaBb\omega|\cos(\omega t)| = 0$$

and recall the previous equation,

$$IBab - T_f = J\alpha$$

Figure 23.9 Calculation of the motor torque (continued)

- Practice problem,

Write the transfer function relating the displacement ’x’ to the current ’I’

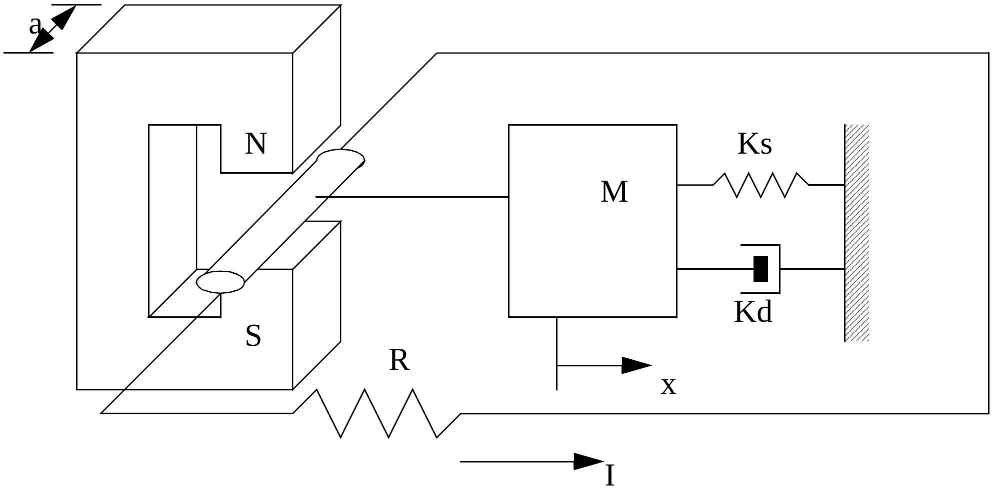


Figure 23.10 Drill problem: Electromotive force

- Consider a motor with a separately excited magnetic field (instead of a permanent magnet there is a coil that needs a voltage to create a magnetic field). The model is similar to the previous motor models, but the second coil makes the model highly nonlinear.

23.4 SUMMARY

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23.5 PRACTICE PROBLEMS

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23.6 PRACTICE PROBLEM SOLUTIONS

23.7 ASSIGNMENT PROBLEMS