

## 28. FUZZY LOGIC

<TODO - Find an implementation platform and add section>

Topics:

- Fuzzy logic theory; sets, rules and solving
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Objectives:

- To understand fuzzy logic control.
- Be able to implement a fuzzy logic controller.

### 28.1 INTRODUCTION

Fuzzy logic is well suited to implementing control rules that can only be expressed verbally, or systems that cannot be modelled with linear differential equations. Rules and membership sets are used to make a decision. A simple verbal rule set is shown in Figure 28.1. These rules concern how fast to fill a bucket, based upon how full it is.

1. If (bucket is full) then (stop filling)
2. If (bucket is half full) then (fill slowly)
3. If (bucket is empty) then (fill quickly)

*Figure 28.1* A Fuzzy Logic Rule Set

The outstanding question is "What does it mean when the bucket is empty, half full, or full?" And, what is meant by filling the bucket slowly or quickly. We can define sets that indicate when something is true (1), false (0), or a bit of both (0-1), as shown in Figure 28.2. Consider the *bucket is full* set. When the height is 0, the set membership is 0, so nobody would think the bucket is full. As the height increases more people think the bucket is full until they all think it is full. There is no definite line stating that the bucket is full. The other bucket states have similar functions. Notice that the *angle* function relates the valve angle to the fill rate. The sets are shifted to the right. In reality this would probably mean that the valve would have to be turned a large angle before flow begins, but after that it increases quickly.

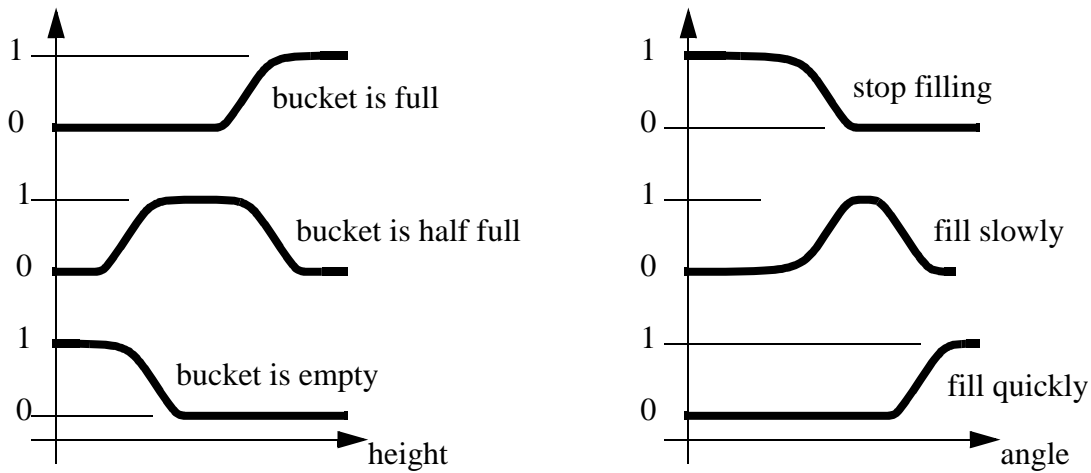


Figure 28.2 Fuzzy Sets

Now, if we are given a height we can examine the rules, and find output values, as shown in Figure 28.3. This begins by comparing the bucket height to find the membership for *bucket is full* at 0.75, *bucket is half full* at 1.0 and *bucket is empty* at 0. Rule 3 is ignored because the membership was 0. The result for rule 1 is 0.75, so the 0.75 membership value is found on the *stop filling* and a value of  $a_1$  is found for the valve angle. For rule 2 the result was 1.0, so the *fill slowly* set is examined to find a value. In this case there is a range where *fill slowly* is 1.0, so the center point is chosen to get angle  $a_2$ . These two results can then be combined with a weighted average to get

$$\text{angle} = \frac{0.75(a_1) + 1.0(a_2)}{0.75 + 1.0}.$$

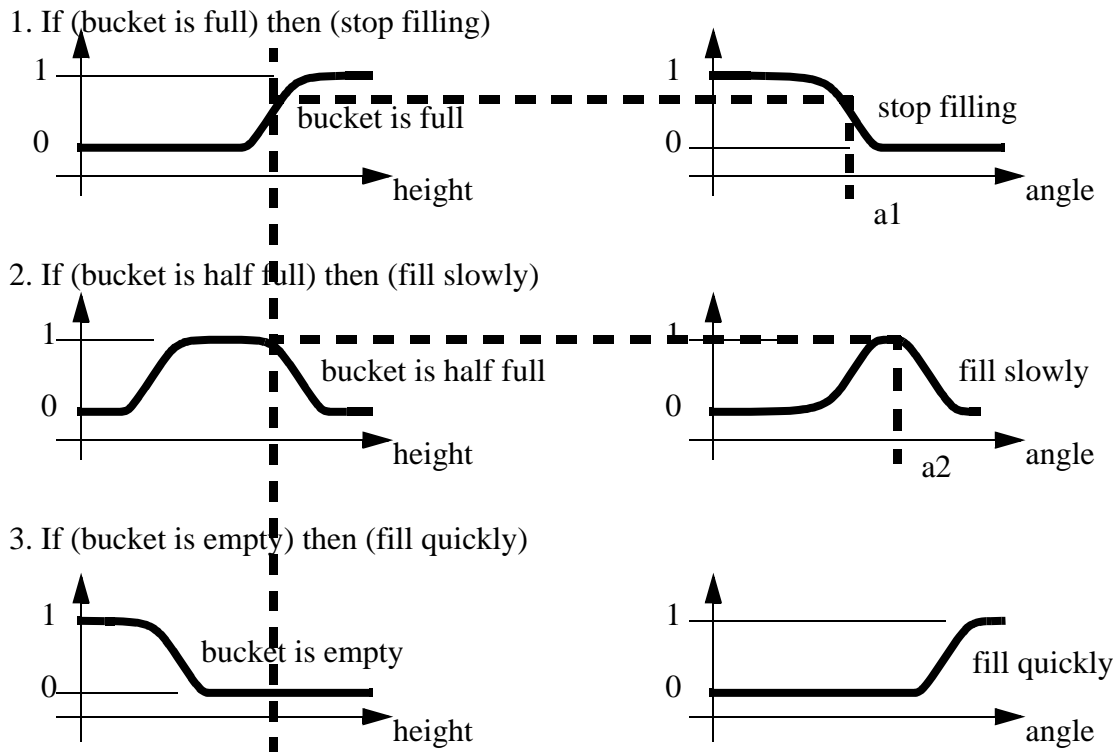
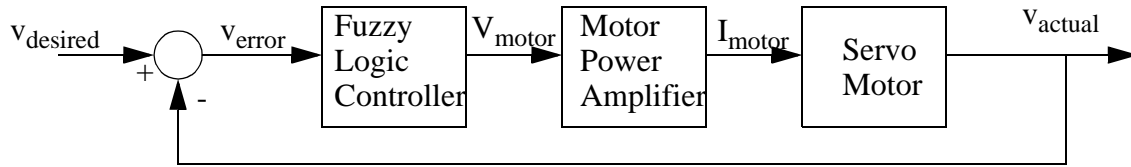


Figure 28.3 Fuzzy Rule Solving

An example of a fuzzy logic controller for controlling a servomotor is shown in Figure 28.4 [Lee and Lau, 1988]. This controller rules examines the system error, and the rate of error change to select a motor voltage. In this example the set memberships are defined with straight lines, but this will have a minimal effect on the controller performance.



The rules for the fuzzy logic controller are;

1. If  $v_{error}$  is LP and  $\frac{d}{dt}v_{error}$  is any then  $V_{motor}$  is LP.
2. If  $v_{error}$  is SP and  $\frac{d}{dt}v_{error}$  is SP or ZE then  $V_{motor}$  is SP.
3. If  $v_{error}$  is ZE and  $\frac{d}{dt}v_{error}$  is SP then  $V_{motor}$  is ZE.
4. If  $v_{error}$  is ZE and  $\frac{d}{dt}v_{error}$  is SN then  $V_{motor}$  is SN.
5. If  $v_{error}$  is SN and  $\frac{d}{dt}v_{error}$  is SN then  $V_{motor}$  is SN.
6. If  $v_{error}$  is LN and  $\frac{d}{dt}v_{error}$  is any then  $V_{motor}$  is LN.

The sets for  $v_{error}$ ,  $\frac{d}{dt}v_{error}$ , and  $V_{motor}$  are;

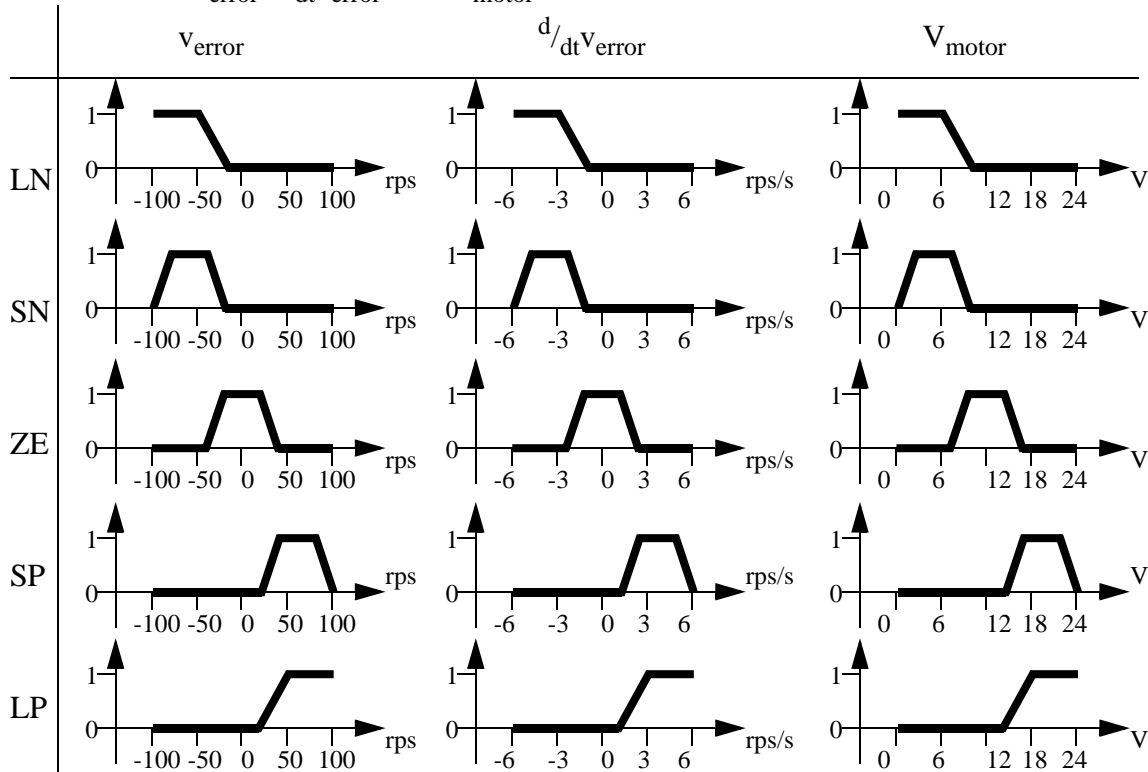
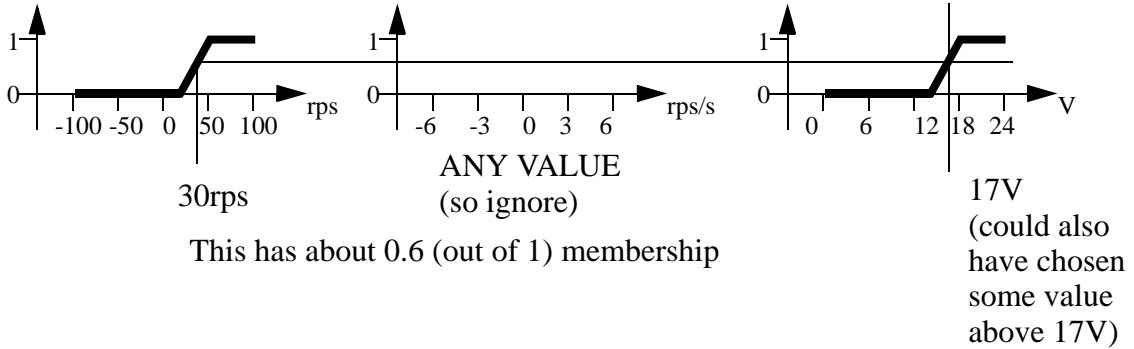


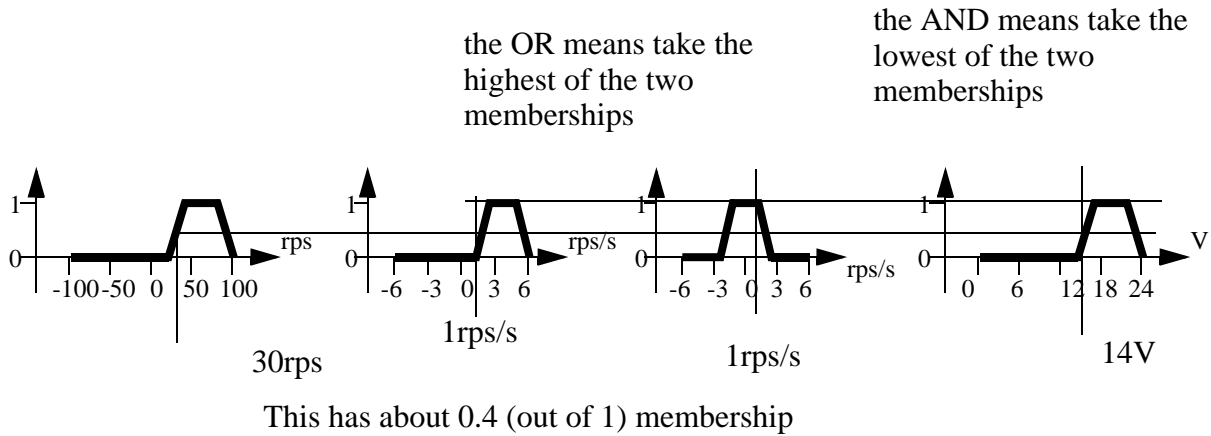
Figure 28.4 A Fuzzy Logic Servo Motor Controller

Consider the case where  $v_{error} = 30$  rps and  $\frac{d}{dt}v_{error} = 1$  rps/s. Rule 1 to 6 are calculated in Figure 28.5.

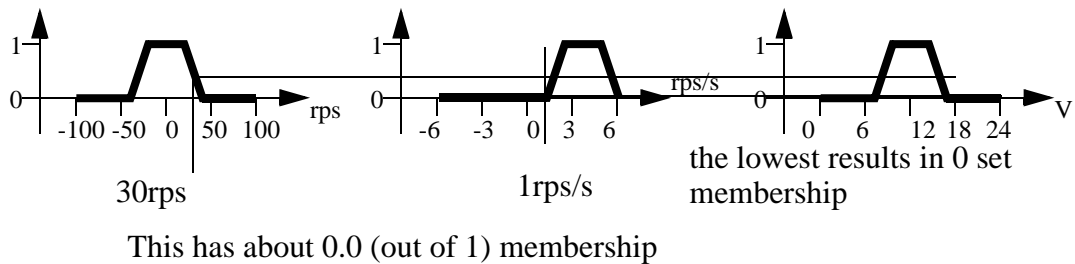
1. If  $v_{error}$  is LP and  $\frac{d}{dt}v_{error}$  is any then  $V_{motor}$  is LP.



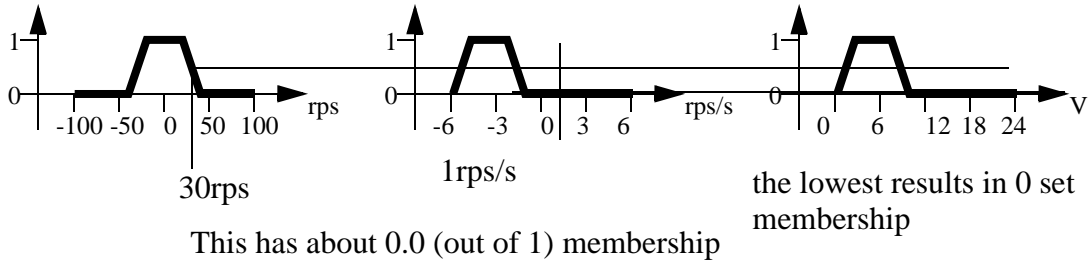
2. If  $v_{error}$  is SP and  $\frac{d}{dt}v_{error}$  is SP or ZE then  $V_{motor}$  is SP.



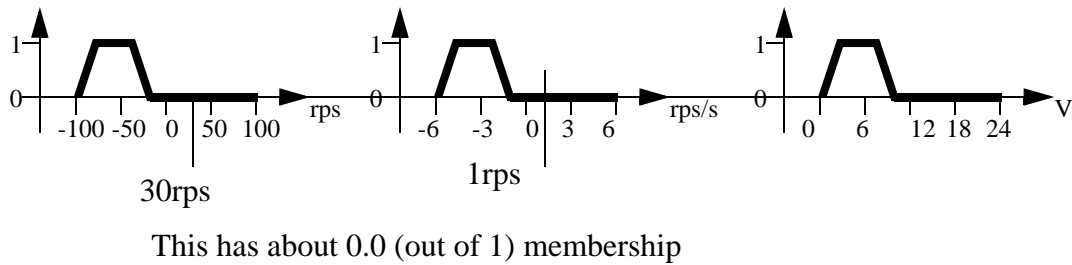
3. If  $v_{error}$  is ZE and  $\frac{d}{dt}v_{error}$  is SP then  $V_{motor}$  is ZE.



4. If  $v_{error}$  is ZE and  $\frac{d}{dt}v_{error}$  is SN then  $V_{motor}$  is SN.



5. If  $v_{error}$  is SN and  $\frac{d}{dt}v_{error}$  is SN then  $V_{motor}$  is SN.



6. If  $v_{error}$  is LN and  $\frac{d}{dt}v_{error}$  is any then  $V_{motor}$  is LN.

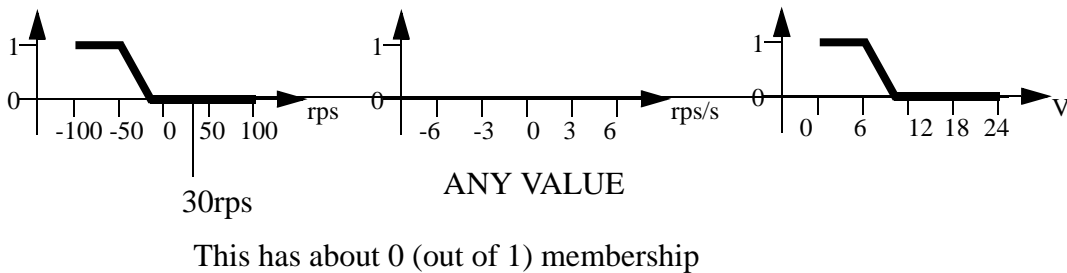


Figure 28.5 Rule Calculation

The results from the individual rules can be combined using the calculation in Figure 28.6. In this case only two of the rules matched, so only two terms are used, to give a

final motor control voltage of 15.8V.

$$V_{motor} = \frac{\sum_{i=1}^n (V_{motor_i})(membership_i)}{\sum_{i=1}^n (membership_i)}$$

$$V_{motor} = \frac{0.6(17V) + 0.4(14V)}{0.6 + 0.4} = 15.8V$$

Figure 28.6 Rule Results Calculation

## 28.2 COMMERCIAL CONTROLLERS

At the time of writing Allen Bradley did not offer any Fuzzy Logic systems for their PLCs. But, other vendors such as Omron offer commercial controllers. Their controller has 8 inputs and 2 outputs. It will accept up to 128 rules that operate on sets defined with polygons with up to 7 points.

It is also possible to implement a fuzzy logic controller manually, possible in structured text.

## 28.3 REFERENCES

Li, Y.F., and Lau, C.C., "Application of Fuzzy Control for Servo Systems", IEEE International Conference on Robotics and Automation, Philadelphia, 1988, pp. 1511-1519.

## 28.4 SUMMARY

- Fuzzy rules can be developed verbally to describe a controller.
- Fuzzy sets can be developed statistically or by opinion.
- Solving fuzzy logic involves finding fuzzy set values and then calculating a value for each rule. These values for each rule are combined with a weighted average.

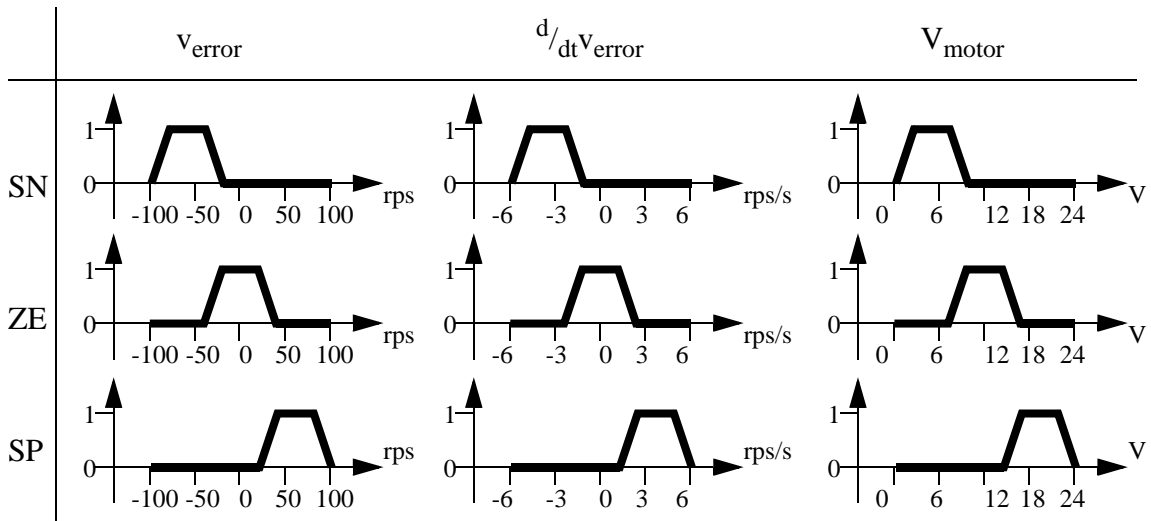
## 28.5 PRACTICE PROBLEMS

## 28.6 PRACTICE PROBLEM SOLUTIONS

## 28.7 ASSIGNMENT PROBLEMS

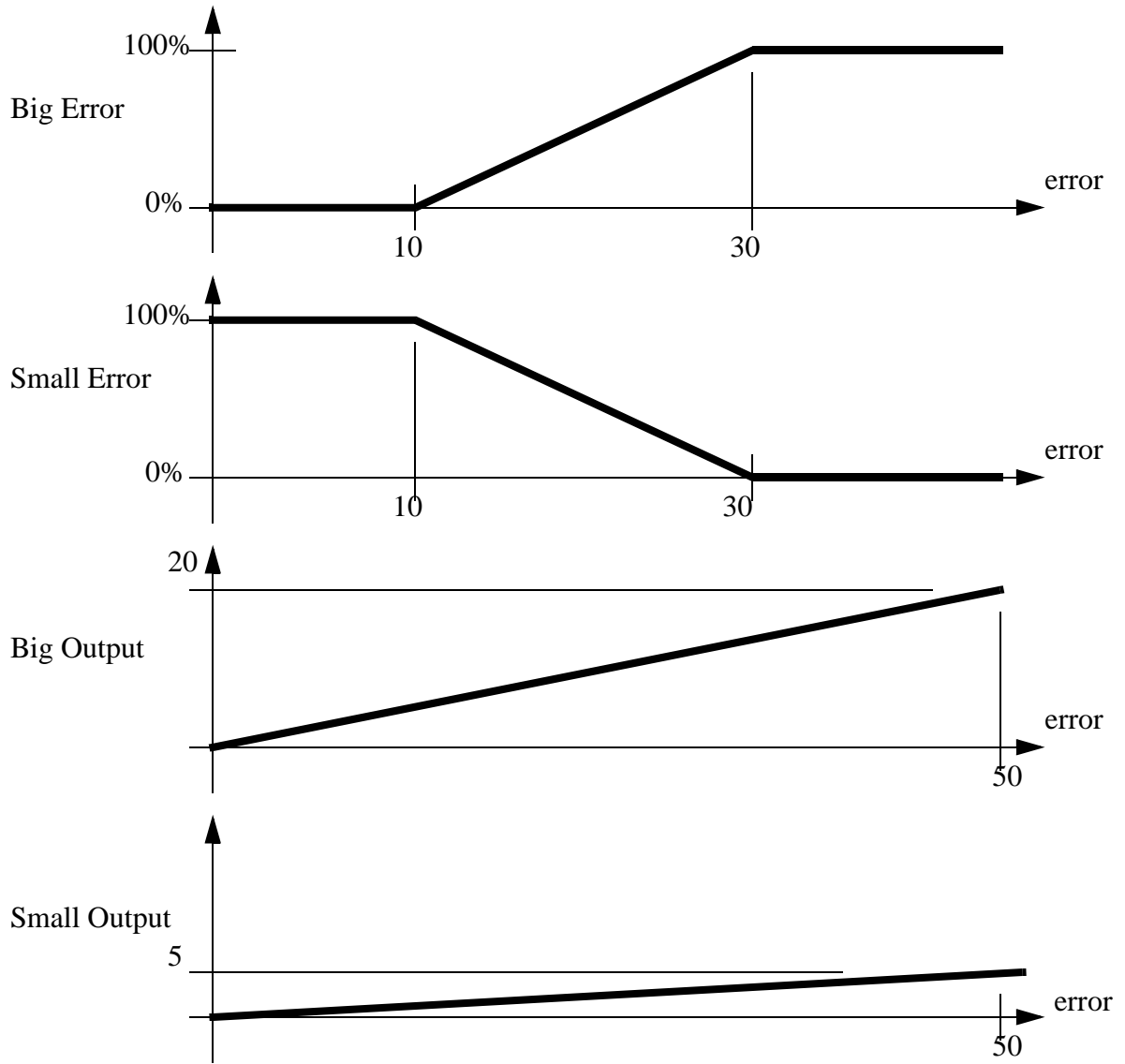
1. Find products that include fuzzy logic controllers in their designs.
2. Suggest 5 control problems that might be suitable for fuzzy logic control.
3. Two fuzzy rules, and the sets they use are given below. If  $v_{\text{error}} = 30\text{rps}$ , and  $\frac{d}{dt}v_{\text{error}} = 3\text{rps/s}$ , find  $V_{\text{motor}}$

1. If ( $v_{\text{error}}$  is ZE) and ( $\frac{d}{dt}v_{\text{error}}$  is ZE) then ( $V_{\text{motor}}$  is ZE).
2. If ( $v_{\text{error}}$  is SP) or ( $\frac{d}{dt}v_{\text{error}}$  is SP) then ( $V_{\text{motor}}$  is SP).



4. Develop a set of fuzzy control rules adjusting the water temperature in a sink.
5. Develop a fuzzy logic control algorithm and implement it in structured text. The fuzzy rule set below is to be used to control the speed of a motor. When the error (difference between desired and actual speeds) is large the system will respond faster. When the difference is smaller the

response will be smaller. Calculate the outputs for the system given errors of 5, 20 and 40.



if (big error) then (big output)  
 if (small error) then (small output)