35. PERMUTATIONS AND COMBINATIONS

Topics:

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Objectives:

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35.1 Introduction

• Permutations - a count of the variety of sequences

• Combinations - a count of the number of selections

• Probability - The chance something will happen.

35.2 Permutations

• the possible arrangements of objects given,

n = number of source objects

r = number of arranged objects

• Permutations - for exact definitions of not only possibilities, but also order.

$$P_r^n = \frac{n!}{(n-r)!}$$

e.g. How many permutations picking 2 cards off a deck (52)

$$\frac{(52)!}{(52-2)!} = \frac{52 \times 51 \times 50 \times \ldots}{50 \times 49 \times \ldots} = 52 \times 51$$

• permutations for n unique objects arranged into r spots

$$\frac{n!}{(n-r)!}$$

For example 5 objects in 3 spots

$$\frac{5!}{(5-3)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 60$$

• permutations for non-unique objects (using all)

For example consider 3 pairs of objects that will fill 6 spots

$$\frac{n!}{d_1!d_2!d_3!} = \frac{6!}{2!2!2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8} = 90$$

35.3 Combinations

• Combinations - similar to before except order does not mater.

$$C_r^n = \frac{n!}{r!(n-r)!} \qquad \text{(n choose r)}$$

e.g. How many combinations of the first 2 cards can be picked off the deck

$$\frac{(52)!}{2!(52-2)!} = \frac{52 \times 51 \times 50 \times \ldots}{(2 \times 1)(50 \times 49 \times \ldots)} = \frac{52 \times 51}{2}$$

• Possible outcomes if ’r’ objects are taken from a group of ’n’.

$$nCr = \frac{nPr}{r!} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}$$

For example consider 2 cars chosen from a set of 4

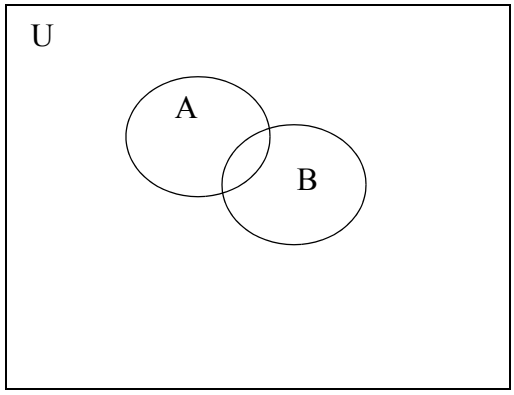
$$\frac{4 \cdot 3}{2 \cdot 1} = 6$$

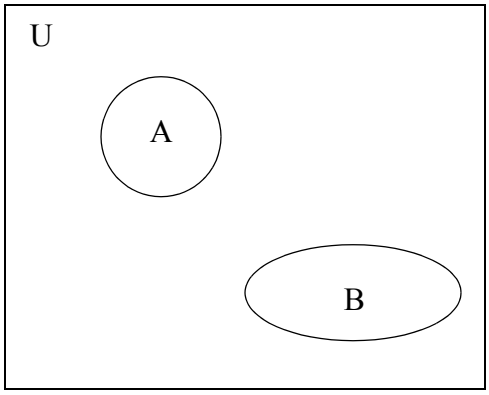
35.4 Probability

• The chance ’P(A)’ some event ’A’ will happen.

• A way to figure out how chances interact

• Venn diagrams can be useful for describing interactions,

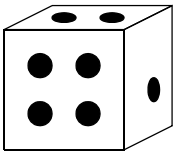


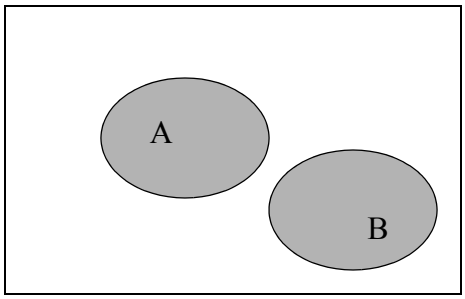


• Mutually Exclusive - Probable events can only happen as one or the other.

e.g. Only one number from 1 to 6 will come up when rolling a die.

$P(A) + P(B) + ... = 1$

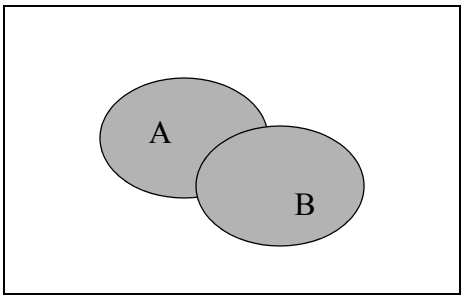




• Not Mutually Exclusive - Probable events can occur simultaneously

e.g. Two dice are rolled to get a number from 2 to 12, the chance that a 4 will come up is,

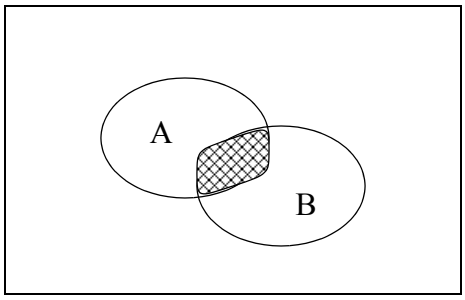
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 1/6 + 1/6 - 1/36$ $P(A \cup B)$



• Independent Probabilities - Events will happen separately

e.g. The chance of rolling a number >= 3 then <3 on a 6 sided die.

P(A and B) = P(A) * P(B) = (4/6)*(2/6) = 8/36 $P(A \cap B)$



• Dependant Probabilities - The outcome of one event effects the outcome of another event

e.g. The chance a student will pass an exam if they show up (99/100) and they know the material (7/10)

P(A and B) = P(A) * P(B when A) = (99/100)*(7/10)

• empirical probability - experimentally determine the probability with,

$$P = \frac{\text{\# of outcomes}}{\text{\# of trials}}$$

• Probability Density Function

$$P(x_1 \le x \le x_2) = \int_{x_1}^{x_2} f(x)dx$$

35.5 Problems

1. The marketing department has asked how many product permutations are possible for a new product. The product will hold three colored balls. In total there are 6 ball colors available. If balls must be used in all spots, and ball colors can be used more than once how many permutations are possible? (ans. 216)

2. A worker is packing small cereal boxes into a larger box. If the worker is picking from a set of 10 different cereal boxes, but can only place 8 in the larger box, how many combinations are possible? (ans. 45)

3. An assembly operation places two parts a board. There is a 3% chance that part A is bad, and a 2% chance that part B is bad. What is the chance the board contains a bad part? (ans. 4.94%)

4. How many ways could 5 operators be assigned to 5 workstations? (ans. 120)

5. How many ways could 8 operators be assigned to 5 workstations? (ans. 6720)

6. There are nine product, each a different color. They are to be put into 3 boxes each holding 3 products. a) How many unique package arrangements are possible? b) How many different combinations of packaged products are possible if the position does not matter? (ans. a) 362,880 b) 1680)

7. An electronics company will assemble circuit boards with interchangeable components. There are 3 places to mount the components and there are 5 types of components. Each component type may only be used once. How many different outcomes are possible? (ans. 60)

8. A toy is being manufactured to have prizes in 2 of 5 slots. How many prize layouts are possible? (ans. 10)

9. Calculate the following values.

a) ${}_7C_3 =$

b) ${}_4C_2 =$

c) ${}_7C_r = 21$

(ans. $= 35$

$= 6$

$r = 2$)

10. There are 8 machines (A to H) waiting to be shipped. 3 of these will be tested.
    a) How many combinations are possible?
    b) How many of those combinations contain machine C?
    c) How many combinations contain A or H, but not both?
    d) If 2 trucks are loaded with 4 machines, how many distributions are possible?
    e) Resolve part d) if machine A and B are in the same truck.
    (ans. a) 56, b) 21, c) 30, d) 70, e) 15)

11. A carton contains 12 parts, 4 are red and 8 are green.
    a) Find the probability that the first part removed is green.
    b) If 3 parts are removed what is the probability that all are red?
    c) Repeat b) for all green.
    d) Repeat b) for one red and 2 green.
    (ans. a) 2/3, b) 1/55, c) 14/55, d) 28/165)

12. Write routines to implement basic functions.