7. OPTIMIZATION

Topics:

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Objectives:

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7.1 Introduction

• This is normally used when there is no clearly optimal solution to a problem.

• The basic procedure is,

1. Idenitfy the major variables in a decision
2. Model the system or decisions to be made with equations
3. Assign a cost (objective) function for the system
4. Select an optimization method and search
5. Analyze the results and search again if necessary

7.2 System Modeling and Variable Identification

• Typical decision variables in systems include,

- mass
- volume
- power consumption
- component cost
- factor of safety
- signal-to-noise ratio

- transmission rates
- etc.

• A model of the system should relate the decisions to be made mathematically. These relationships oftem include performance measures,
  - cutting speeds
  - cycle times
  - power
  - capacity
  - flow rates
  - etc.

7.3 Cost Functions and Constraints

• Cost functions quantify things that we want to minimize, or maximize. These should be aligned with system objectives. Typical cost functions include,

- money
- time
- some combination of factors

• Systems also have constraints that limit available solutions

• Expressed as a function of variables that provides a value

• Consider the example of building a fenced pasture. In this case when the area becomes too large, there is a reduced value. We want to maximize the value of V.

$$C_{fence} = 20\left(\frac{\$}{m}\right)(2w + 2d)$$

where,

C_{fence} = cost to construct the fence ($)

w = width of the pasture (m)

d = depth of the pasture (m)

$$C_{land} = 0.35\left(\frac{\$}{m^2}\right)$$

where,

C_{land} = cost for the land

$$R = 0.05\left(\frac{\$}{m^2 yr}\right)wd \qquad wd < 300m^2$$

$$R = 0.01\left(\frac{\$}{m^2 yr}\right)(wd - 300m^2) + 60\$ \qquad wd \geq 300m^2$$

where,

R = revenue generated by pasture land

$$V = R - C_{fence} - C_{land}$$

where,

V = total value

Figure 1.29 Example cost function for building a fence around a pasture

- The cost function can be written as..

```
double cost(double w, double d){
	double value;
	double cfence, cland, R;

	Cfence = 40*(w + d);
	Cland = 0.05*w*d;
	if (w*d < 300){
				R = 0.05 * w * d;
	} else {
				R = 0.01 * (w * d - 300) + 60;
	}
	value = R - Cfence - Cland

	return value;
}
```

Figure 1.30 A subroutine for cost function calculation

• Constraints are boundaries that cannot be crossed.

• Example of constraints, the pasture cannot be larger than one 1600m be 1600m beacuse of the constraints of an existing road system.

$$w \leq 1600 m$$

$$d \leq 1600 m$$

Figure 1.31 Example constraint functions for a pasture

• The cost function can be written as..

```
double cost(double w, double d){
        double value;
        double cfence, cland, R;

        Cfence = 40*(w + d);
        Cland = 0.05*w*d;
        if (w*d < 300){
                        R = 0.05 * w * d;
        } else {
                        R = 0.01 * (w * d - 300) + 60;
        }
        value = R - Cfence - Cland

        if(w > 1600) value = 1000000;
        if(d > 1600) value = 1000000;

        return value;
}
```

Figure 1.32 A subroutine for cost function calculation

• Slack variables allow constraints to be considered as part of the cost function. Helps with a system with many local minimum.

$$C_{penalty} = \left(\frac{w}{1600m}\right)^4 + \left(\frac{d}{1600m}\right)^4$$

$$V = R - C_{fence} - C_{land} - C_{penalty}$$

Figure 1.33 Example of slack variables for including constraints

• The cost function can be written as..

```
double cost(double w, double d){
	double value;
	double cfence, cland, R;
	double slack;

	Cfence = 40*(w + d);
	Cland = 0.05*w*d;
	if (w*d < 300){
				R = 0.05 * w * d;
	} else {
				R = 0.01 * (w * d - 300) + 60;
	}
	// slack variable
	slack = pow((w/1600), 4) + pow((d/1600), 4);
	value = R - Cfence - Cland - slack;

	return value;
}
```

Figure 1.34 A subroutine for cost function calculation

7.4 Single Variable Searches

• For simple single value problems use derivatives and find the maxima/minima.

7.4.1 Example Problem

• As with most engineering problems we want to get the highest return, with the minimum investment. In this case we want to minimize costs, while increasing cutting speeds.

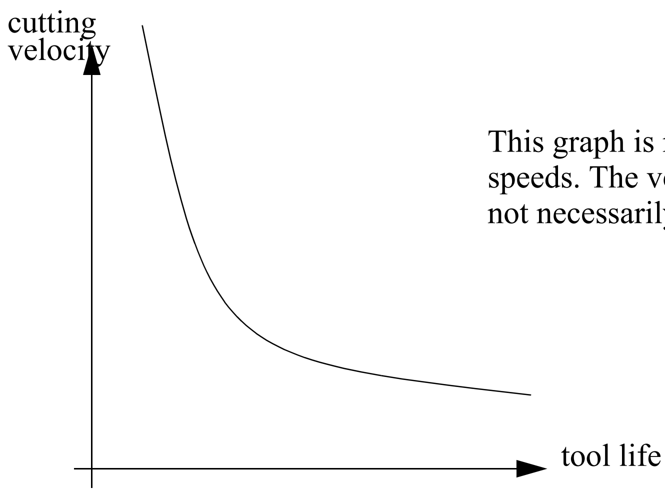
• EFFICIENCY will be the key term - it suggests that good quality parts are produced at reasonable cost.

• Cost is a primarily affected by,
  - tool life
  - power consumed

• The production throughput is primarily affected by,
  - accuracy including dimensions and surface finish
  - mrr (metal removal rate)

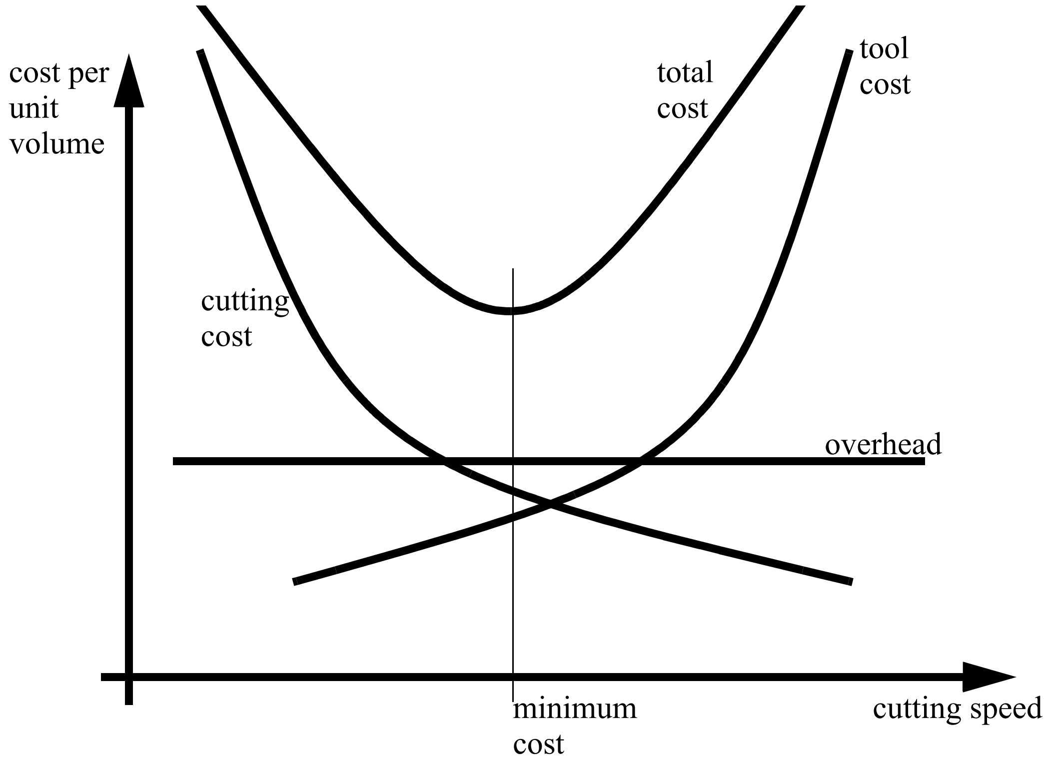
• The factors that can be modified to optimize the process are,
- cutting velocity (biggest effect)
- feed and depth
- work material
- tool material
- tool shape
- cutting fluid

• We previously considered the log-log scale graph of Taylor's tool life equation, but we may also graph it normally to emphasize the effects.



This graph is representative for most reasonable cutting speeds. The velocities at the high and low ranges do not necessarily exhibit the same relationship.

• There are two basic conditions to trade off,
- Low cost - exemplified by low speeds, low mrr, longer tool life
- High production rates - exemplified by high speeds, short tool life, high mrr
*** There are many factors in addition to these, but these are the most commonly considered



• A simplified treatment of the problem is given below for optimizing cost, First lets look at costs for a cutting tool over the life of a tool,

$$C_t = c_1 + c_2 + c_3$$

where,

C_t = cost per cutting edge
c_1 = the cost to change a tool
c_2 = the cost to grind a tool per edge
c_3 = the cost of the tool per edge

and,

$$c_1 = t_1 \times R_c$$

$$c_2 = t_2 \times \frac{R_s}{N_1}$$

$$c_3 = \frac{C_T}{N_1 \times (N_2 + 1)}$$

where,

t_1 = tool change time
t_2 = tool grind time in minutes
R_c = cutting labour + overhead cost
R_s = grinding labor + overhead cost
C_T = cost of the original tool
N_1 = the number of cutting edges to grind
N_2 = the maximum number of regrinds

and,

$$C_c = R_c \times T$$

where,

C_c = cutting operation cost over life of tool, per edge
T = tool life

Next, lets consider the effects of metal removal rate,

$$Q_T = V \times T \times f \times c \qquad (1)$$

where,

Q_T = metal removal rate per edge
V = cutting velocity
f = tool feed rate
c = depth of width of the cut

consider the life of the tool,

$$V \times T^n = C \text{ (Taylors tool life equation)}$$

$$\therefore V = \frac{C}{T^n} \qquad (2)$$

Now combine tool life (2) with the mrr (1),

$$Q_T = V \times T \times f \times c = \frac{C}{T^n} \times T \times f \times c = \frac{C \times f \times c}{T^{n-1}}$$

At this point we have determined functions for cost as a function of tool life, as well as the metal removal rates. We can now proceed to find cost per unit of material removed.

$$C_u = \frac{C_c + C_t}{Q_T} = \frac{T^{n-1}}{C \times f \times c}(R_c \times T + C_t)$$

Using some basic calculus, we can find the minimum cost with respect to tool life.

$$\frac{dC_u}{dT} = \left(\frac{1}{C \times f \times c}\right)(R_c \times n \times T^{n-1} + C_t \times (n-1) \times T^{n-2}) = 0$$

$$\therefore R_c \times n \times T = -C_t \times (n-1)$$

$$\boxed{\therefore T = \frac{-C_t \times (n-1)}{R_c \times n} = \frac{C_t}{R_c}\left(\frac{1-n}{n}\right)}$$

• We can also look at optimizing production rates, There are two major factors here when trying to increase the mrr. We can have a supply of tools by the machine, and as the tools require replacement, the only down-time involved is the replacement of the tool.

This gives us an average rate of production,

$$R_p = \frac{Q_T}{T + t_1}$$

where,

R_p = average rate of production

recall from before that,

$$Q_T = \frac{Cfc}{T^{n-1}}$$

now substituting in gives,

$$R_p = \frac{\left(\frac{Cfc}{T^{n-1}}\right)}{T + t_1} = Cfc(T^n + t_1)^{-1}$$

We can now optimize the production rate,

$$\frac{dR_p}{dT} = Cfc[-(T^n + t_1)^{-2} + (nT^{n-1} + t_1)(T^n + t_1)^{-1}] = 0$$

$$\therefore (T^n + t_1)^{-2} = (nT^{n-1} + t_1)(T^n + t_1)^{-1}$$

$$\therefore 1 = (nT^{n-1} + t_1)(T^n + t_1)$$

$$\therefore 1 = nT^{2n-1} + nt_1T^{n-1} + t_1T^n + t_1^2$$

$$\therefore \log(1) = \log(nT^{2n-1}) + \log(nt_1T^{n-1}) + \log(t_1T^n) + \log(t_1^2)$$

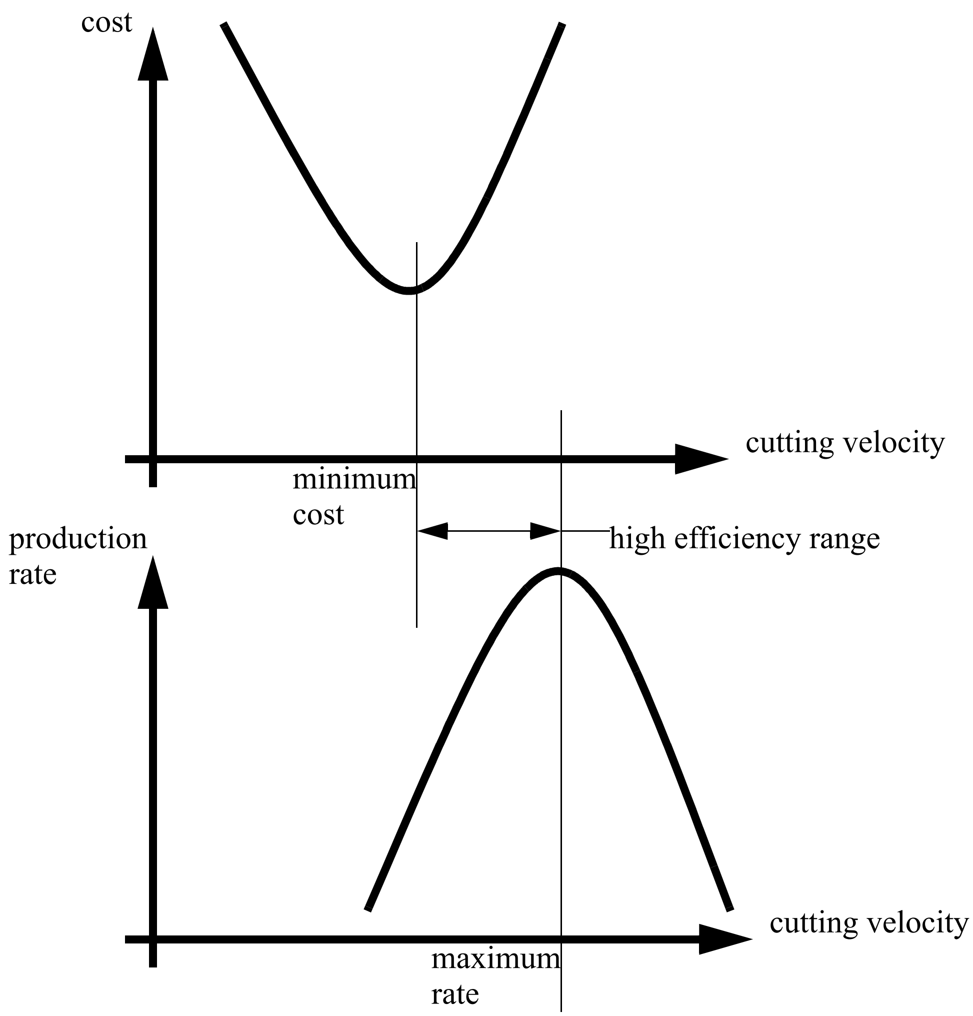
$$\therefore 0 = \log(n) + (2n-1)\log(T) + \log(nt_1) + (n-1)\log(T) + \log(t_1) + n\log(T) + \log(t_1^2)$$

$$\therefore 0 = 2\log(n) + (4n-2)\log(T) + 4\log(t_1)$$

$$\boxed{\therefore \log(T) = \frac{\log(n) + 2\log(t_1)}{1 - 2n}}$$

• We can now put the two optimums in perspective,

Since, $t_1 < C_t/R_c$ then tool life for maximum production is less than economical tool life and as a result, cutting velocity for maximum production is > velocity for lowest cost



7.5 Multivariable Searches

• Local Search Space

• A topographical map shows the relationship between search parameters and cost values.

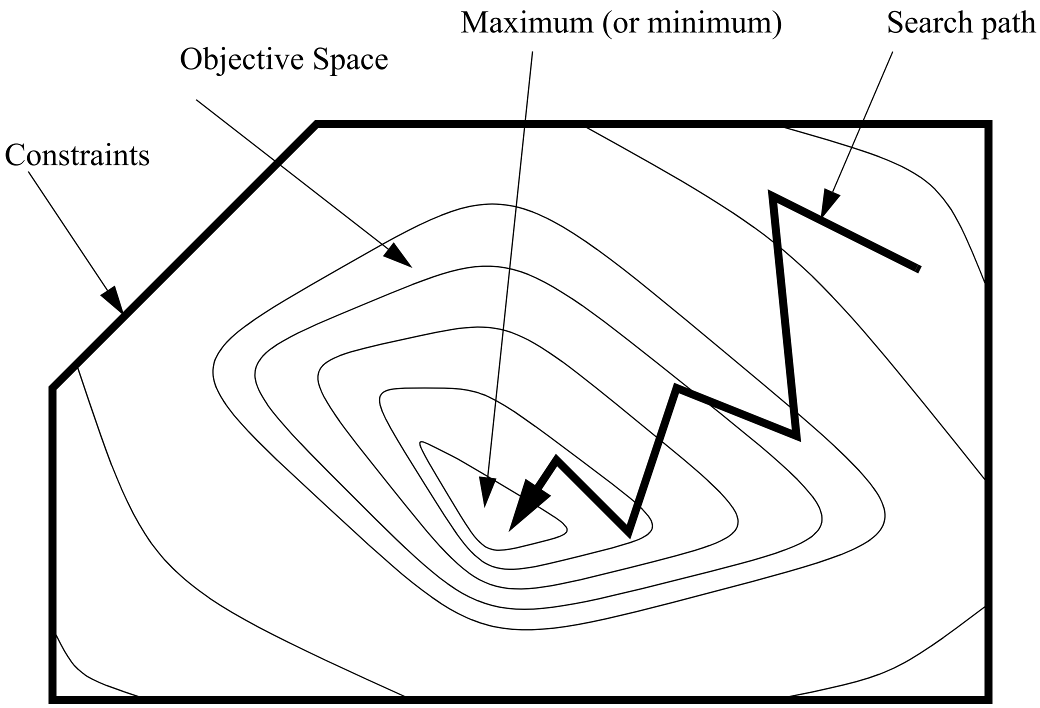


Figure 1.35 Local searches

• Global Search space. In this case the system becomes 'stuck' in a local mimina.

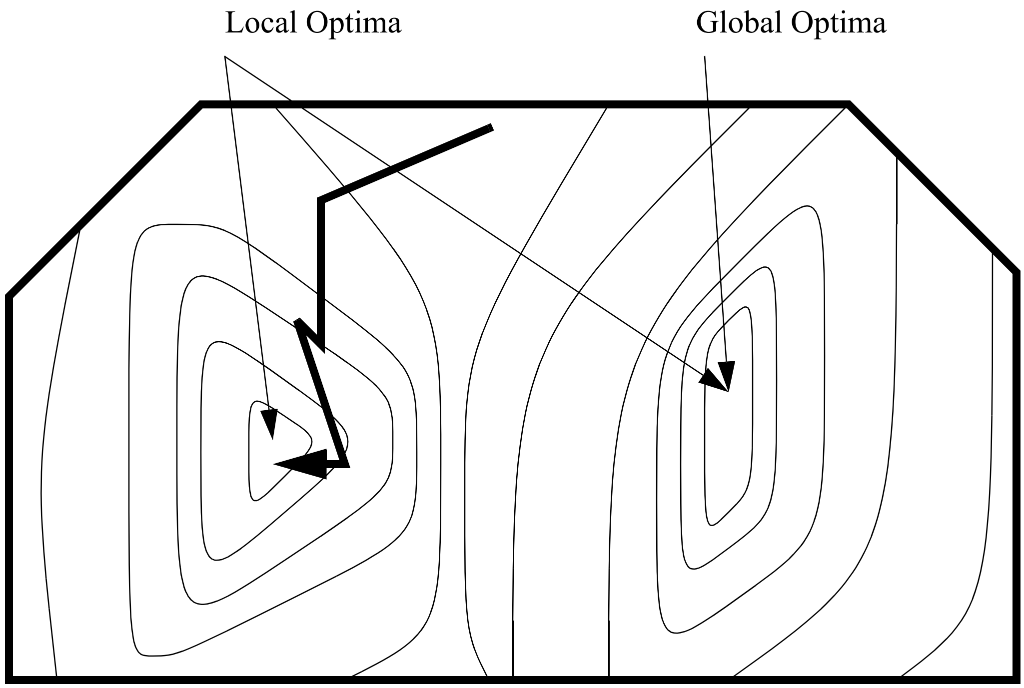


Figure 1.36 Global searches

• Global Search space. In this case the system searches all maxim.

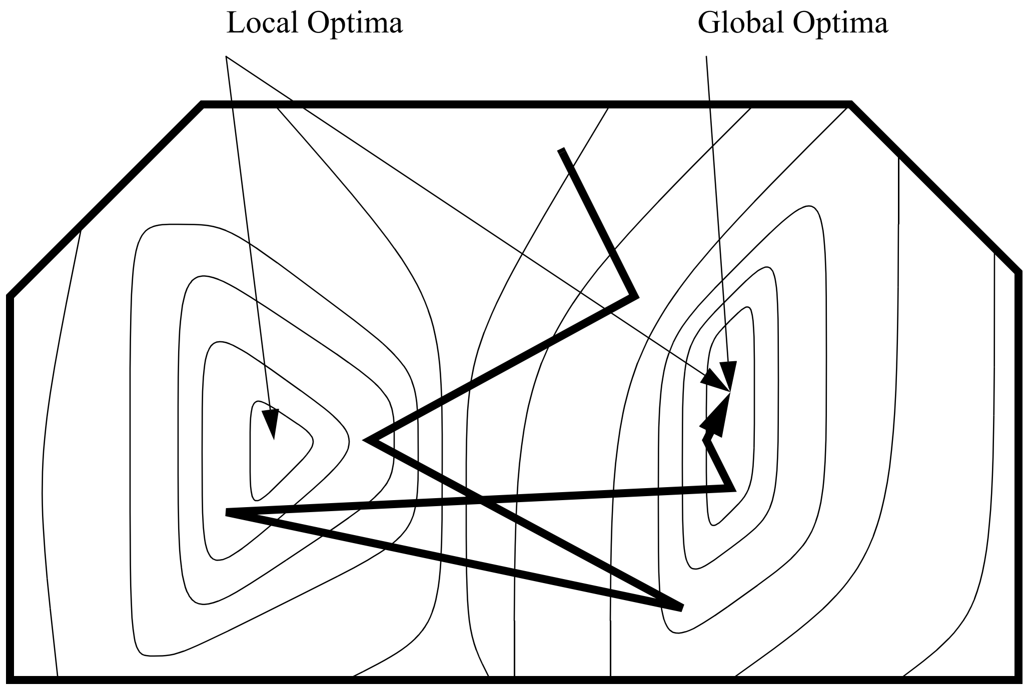


Figure 1.37 A Global Search

7.5.1 Algorithms

• The search algorithms change system parameters and try to lower system parameters.

• The main question is how to change the system paramters to minimize the system value.

7.5.2 Random Walk

7.5.3 Gradient Decent

7.5.4 Simplex

1.6 Summary

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1.7 Problems

1. A production facility molds plastic parts. Ideally these parts are made with new plastic pellets. However rejected plastic parts are ground into (regrind) pellets and mixed in with new plastic pellets. New plastic costs $1.00 per pound. If a part is discarded (not reground) the total cost of the material is lost. To regrind and dry scrap parts there is a cost of $0.10 per pound. The customer demands that the percentage of regrind cannot exceed 30%. Statistical data was used to find the following relationship between the percentage of regrind and the scrap rate. Write a program to determine the optimum quantity of regrind to minimize the cost per part.

$$S = 2.0 + 3.0 \times 2^{\frac{R+5}{4}}$$

where,

$R =$ percentage of reground material [0, 100%]

$S =$ percentage of parts that are scrap

1.8 Challenge Problems

1. Rocket Fuel Burn

2. Power plant fuel mixture

3. Write a program that recommends the optimum cutting speed for a machining process. Model the program on the example in the chapter.