28. NUMERICAL VALUES AND UNITS

Topics:
- Numbers, Constants, Units

Objectives:
- To review the use of basic numbers, including significant figures
- To review the use of units to keep track of numerical magnitudes

28.1 Introduction

28.2 Numerical Values

28.2.1 Constants and Other Stuff

• Some basic definitions,

numeric - a literal numerical value

variable - a symbol used to represent a quantity that will change, often represented with a lower case symbol

constant - a value that will not change, often represented with an upper case symbol

subscripts - letters or numbers below a variable to create new (related) variables.

• greek letters are often used for variables and constants

lower case	upper case	name
α	Α	alpha
β	Β	beta
γ	Γ	gamma
δ	Δ	delta
ε	Ε	epsilon
ζ	Ζ	zeta
η	Η	eta
θ	Θ	theta
ι	Ι	iota
κ	Κ	kappa
λ	Λ	lambda
μ	Μ	mu
ν	Ν	nu
ξ	Ξ	xi
ο	Ο	omicron
π	Π	pi
ρ	Ρ	rho
σ	Σ	sigma
τ	Τ	tau
υ	Υ	upsilon
φ	Φ	phi
χ	Χ	chi
ψ	Ψ	psi
ω	Ω	omega

• The constants listed are some of the main ones, other values can be derived through calculation.

$$e = 2.7182818\ldots = \lim_{n \to \infty}\left(1 + \frac{1}{n}\right)^n = \text{natural logarithm base}$$

$$\pi = 3.1415927\ldots = \text{pi}$$

$$\gamma = 0.57721566 = \text{Eulers constant}$$

$$1\,radian = 57.29578°$$

• In Scilab

```
// basic variables and constants
a = 5; // define a variable with a value of 5
b = %pi; // the value for pi is assigned to b
c = %e; // the natural number
d = %inf; // infinity
e = %nan; // not a number
m = %t; // a logical true
n = %f; // a logical false
p = %i; // the imaginary number
q = eps; // a very small positive number, or 0+

abs(x); // returns the magnitude of x
int(x); // converts a real to an integer value


// getting information
who // print all variables
help sin // open a help window for the sin function
help + // get help on basic operators
apropos imaginary // look for functions on imaginary numbers
```

28.2.2 Factorial

• A compact representation of a series of increasing multiples.

$$n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots \cdot n$$

$$0! = 1$$

28.2.3 Significant Figures

• Sig figures rules,

- leading zeroes do not count as significant figures.
- trailing zeros will count as significant figures.
- when doing multiplying the results should will (generally) have the same number of significant figures as the least significant number.
- when adding, the least accurate number determines the accuracy of the result.

123.456	6 significant figures
123.45600	8 significant figures
0.000345	3 significant figures
$123 \cdot 456789 = 56185047 = 56.2 \times 10^3$	3 significant figures
$0.12(3456) = 414.72 = 0.41 \times 10^3$	2 significant figures
$34 + 56.789 = 90.789 = 91$	2 significant figures

• In computation the standard is to keep all of the digits, but the final answer should be rounded to the correct number of significant figures

• Based upon the accuracy of most measuring instruments, and the ability to specify components, most engineering calculations will have 3-6 significant figures. Do not use all of the digits produced by computer/calculator unless all of the digits can be justified.

28.2.4 Scientific and Engineering Notations

• In scientific notation one digit is ahead of the decimal, and all other values follow the decimal. The exponent is adjusted accordingly.

$$a = 1234.56789 = 1.23456789 \times 10^3 = 1.23456789e3$$

• Scilab

```
a = 1234.5678912345678;
a // by default 8 digits will be printed
format('v', 20) // set the number of displayed characters to 20
a // prints 1234.567891234567892
format('v', 5)
a // prints 1234.
format('e', 8) // set the display to exponent notation
a // prints 1.2D+03
a = 1.23456789123456789e3; // the same value of a entered in exponent format
```

• Engineering notation is similar to scientific notation, but the exponent is always a multiple of 3 so that it corresponds to magnitude multipliers (i.e., micro, milli, kilo, mega).

$$a = 12345.6789 = 12.3456789e3$$

$$b = 0.000123456789 = 123.456789e-6$$

• The current version of Scilab does not seem to support engineering notation.

28.3 Complex Numbers

• 'j' will be the preferred notation for the complex number, this is to help minimize confusion with the 'i' used for current in electrical engineering.

• The basic algebraic properties of these numbers are,

The Complex (imaginary) Number:

$$j = \sqrt{-1} \qquad j^2 = -1$$

• Scilab,

```
j = sqrt(-1); // define j as the imaginary number
A = 5 + 3*j; // define a complex number
B = 7 + 9j; // define another
A / B // a complex operation
```

28.4 Units and Conversions

• Units are essential when describing real things.

• Good engineering practice demands that each number should always be accompanied with a unit.

28.4.1 How to Use Units

• This section does not give an exhaustive list of conversion factors, but instead a minimal (but fairly complete) set is given. From the values below most conversion values can be derived.

• A simple example of unit conversion is given below,

**a simple unit conversion example:

Given,

$$d_x = 10m \qquad d_y = 5ft$$

Find the distance 'd',

$$d = \sqrt{d_x^2 + d_y^2}$$

keep the units in the equation

$$\therefore d = \sqrt{(10m)^2 + (5ft)^2}$$

$$\therefore d = \sqrt{100m^2 + 25ft^2}$$

multiply by 1

$$\therefore d = \sqrt{100m^2 + 25ft^2\left(\frac{0.3048m}{1ft}\right)^2}$$

$$\therefore d = \sqrt{100m^2 + 25ft^2(0.092903)\frac{m^2}{ft^2}}$$

cancel out units

$$\therefore d = \sqrt{100m^2 + 25(0.092903)m^2}$$

$$\therefore d = \sqrt{102.32m^2} = 10.12m$$

From the tables

$$1ft = 0.3048m$$

$$\therefore 1 = \frac{0.3048m}{1ft}$$

28.4.2 SI Units

1. Beware upper/lower case letter in many cases they can change meanings.
   e.g. m = milli or mega?
2. Try to move prefixes out of the denominator of the units.
   e.g., N/cm or KN/m
3. Use a slash or exponents.
   e.g., (kg•m/s^2) or (kg•m•s^{-2}) or (kg m s**-2) or (kg m s ^-2)
4. Use a dot in compound units when possible.
   e.g., N•m
5. Use spaces to divide digits when there are more than 5 figures, commas are avoided because their use is equivalent to decimal points in many places (e.g., Europe).

• Base and derived units

Base units

m = length

kg = mass

s = time

A = current

K = temerature

mol = chemical quantity

cd = candela

Derived unit examples

$N = \frac{kg \cdot m}{s^2}$ $F = \frac{C}{V}$

$J = N \cdot m$ $\Omega = \frac{V}{A}$

$Pa = \frac{N}{m^2}$ $Wb = V \cdot s$

$W = \frac{J}{s}$ $T = \frac{Wb}{m^2}$

$V = \frac{W}{A}$ $H = \frac{Wb}{A}$

• In some cases units are non-standard. There are two major variations US units are marked with 'US' and Imperial units (aka English and inch based) are marked with 'IMP'.

28.4.3 A Table

Distance

1 ft. (feet) = 12 in. (inches) = 0.3048 m (meter)
1 mile = 1760 yards = 5280 ft = 1.609km
1 in.(inch) = 2.540 cm
1 yd (yard) = 3 ft.
1 nautical mile = 6080 ft. = 1852 m = 1.150782 mi
1 micron = 10^{-6} m
1 angstrom = 10^{-10} m

1 mil = 10^{-3} in
1 acre = 43,560 ft. = 0.4047 hectares
1 furlong = 660 ft
1 lightyear = 9.460528e15 m
1 parsec = 3.085678e16 m

Area

1 acre = 43,559.66 ft^2
1 Hectare (ha) = 10,000 m^2
1 Hectare (ha) = 10,000 m^2
1 Hectare (ha) = 10,000 m^2
1 Hectare (ha) = 10,000 m^2

Velocity

1 mph = 0.8689762 knot

Angle

1 rev = 2PI radians = 360 degrees = 400 gradians
1 degree = 60 minutes
1 minute = 60 seconds

Volume

1 US gallon = 231 in^3
1 CC = 1 cm^3
1 IMP gallon = 277.274 in^3
1 barrel = 31 IMP gal. = 31.5 US gal.
1 US gal. = 3.785 l = 4 quarts = 8 pints = 16 cups
1 liter (l) = 0.001 m^3 = 2.1 pints (pt) = 1.06 quarts (qt) = 0.26 gallons (gal)
1 qt (quart) = 0.9464 l
1 cup (c) = 0.2365882 l = 8 USoz
1 US oz = 8 US drams = 456.0129 drops = 480 US minim = 1.0408 IMP oz
= 2 tablespoons = 6 teaspoons
1 IMP gal. = 1.201 U.S. gal.
1 US pint = 16 US oz
1 IMP pint = 20 IMP oz
1tablespoon = 0.5 oz.
1 bushel = 32 quarts
1 peck = 8 quarts

Force/Mass

1 N (newton) = 1 kg•m/s^2 = 100,000 dyne
1 dyne = $2.248*10^{-6}$ lb. (pound)
1 kg = 9.81 N (on earth surface) = 2.2046 lb

1lbf = 16 oz. (ounce) = 4.448N
1 oz. = 28.35 g (gram) = 0.2780N
1 lb = 0.03108 slug
1 kip = 1000 lb.
1 slug = 14.59 kg
1 imperial ton = 2000 lb = 907.2 kg
1 metric tonne = 1000 kg
1 troy oz = 480 grain (gr)
1 g = 5 carat
1 pennyweight = 24 grain
1 stone = 14 lb
1 long ton = 2240 lb
1 short ton = 2000 lb

Pressure

1 Pascal (Pa) = 1 N/m^2 = 6.895 kPa
1 atm (metric atmos.) =760 mmHg at 0°C=14.223 lb/in^2=$1.0132*10^5$ N/m^2
1 psi = 2.0355 in. Hg at 32F = 2.0416 in. Hg at 62F
1 microbar = 0.1 N/m^2

Scale/Magnitude

atto (a) = 10^{-18}
femto (f) = 10^{-15}
pico (p) = 10^{-12}
nano (n) = 10^{-9}
micro (μ) = 10^{-6}
milli (m) = 10^{-3}
centi (c) 10^{-2}
deci (d) = 10^{-1}
deka (da) = 10
hecto (h) = 10^2
kilo (K) = 10^3
mega (M) = 10^6
giga (G) = 10^9
tera (T) = 10^{12}
peta (P) = 10^{15}
exa (E) = 10^{18}

Power

1 h.p. (horsepower) = 745.7 W (watts) = 2.545 BTU/hr. = 550 ft.lb./sec.
1 ft•lb/s = 1.356 W
1 J (joule) = 1 N•m = 10^7 ergs = 0.2389 cal.

1 W = 1 J/s
1 ev = $1.60219*10^{-19}$ J
1 erg = 10^{-7} J

Temperature

°F = [(°C*9)/5]+32, °C = Celsius (Centigrade), F = Fahrenheit
K = Kelvin
Rankine (R) = F - 459.666
0.252 calories = 1 BTU (British Thermal Unit)
-273.2 °C = -459.7 °F = 0 K = 0 R = absolute zero
0 °C = 32 °F = 273.3 K = 491.7 R = Water Freezes
100°C = 212°F = 373.3 K = 671.7 R = Water Boils (1 atm. pressure)
1 therm = 100,000 BTU

Mathematical

π radians = 3.1416 radians = 180 degrees = 0.5 cycles
1 Hz = 1 cycle/sec.
1 rpm (revolutions per minute) = 60 RPS (Revolutions per second) = 60Hz
1 fps (foot per second) = 1 ft/sec
1 mph (miles per hour) = 1 mi./hr.
1 cfm (cubic foot per minute) = 1 ft^3/min.
e = 2.718

Time

1 Hz (hertz) = 1 s^{-1}
1 year = 365 days = 52 weeks = 12 months
1 leap year = 366 days
1 day = 24 hours
1 fortnight = 14 days
1 hour = 60 min.
1 min = 60 seconds
1 millenium = 1000 years
1 century = 100 years
1 decade = 10 years

Physical Constants

R = 1.987 cal/mole K = ideal gas law constant
K = Boltzmann's constant = $1.3x10^{-16}$ erg/K = $1.3x10^{-23}$ J/K
h = Planck's constant = $6.62x10^{-27}$ erg-sec = $6.62x10^{-34}$ J.sec
Avagadro's number = $6.02x10^{23}$ atoms/atomic weight
density of water = 1 g/cm^3
electron charge = $1.60x10^{-19}$ coul.
electron rest mass = 9.11*10**-31 Kg

proton rest mass = 1.67*10**-27 Kg
speed of light (c) = 3.00x10^{10} cm/sec
speed of sound in dry air 25 C = 331 m/s
gravitational constant = 6.67*10**-11 Nm**2/Kg**2
permittivity of free space = 8.85*10**-12 farad/m
permeability of free space = 1.26*10**-6 henry/m
mean radius of earth = 6370 Km
mass of earth = 5.98*10**24 Kg

Electromagnetic

magnetic flux = weber (We) = 10**8 maxwell
inductance = henry
magnetic flux density = tesla (T) = 10**4 gauss
magnetic intensity = ampere/m = 0.004*PI oersted
electric flux density = coulomb/m**2
capacitance = farad
permeability = henry/m
electric field strength = V/m
luminous flux = lumen
luminance = candela/m**2
1 flame = 4 foot candles = 43.05564 lux = 43.05564 meter-candles
illumination = lux
resistance = ohm

28.5 Problems

1. Show the units for Joules in base units. (ans. kg m^2 / s^2)

2. How many kips are in 2.00 metric tonnes. (ans. 4.41 kip)

3. Do the following calculation using significant figures.

$$0.010 \times 1.2345 + 0.0234567$$ (ans. 35e-3)

2. Simplify the following expressions.

a) $(6 + 8j)^2$ ans. $-28 + 96j$

b) $\dfrac{8j + 6}{(4j + 3)^2}$ ans. $\dfrac{6}{25} - \dfrac{8}{25}j$

c) $2 + 5!$ ans. 122